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PROBLEM SET 07

PROBLEM 1

Suppose a researcher wants to estimate the parameters in the simple population equation

$$Y = \beta_0 + \beta_1 X + u$$

The following data which were calculated from a sample of 10 observations are available.

$$\begin{aligned}\Sigma Y &= 580 & \Sigma xy &= -654 \\ \Sigma X &= 50 & \Sigma y^2 &= 7568 \\ \Sigma XY &= 2246 & \Sigma x^2 &= 60 \\ \Sigma Y^2 &= 41208 \\ \Sigma X^2 &= 310\end{aligned}$$

- Obtain least square estimates of β_0 and β_1 .
- Compute R^2 . Given an interpretation for the coefficients of determination.
- Test if the model must have an intercept term (take $\alpha = 0.05$).
- Test if the slope coefficient is different from zero (take $\alpha = 0.05$).
- Construct the 99 % confidence interval for β_1 .

PROBLEM 2

Suppose you want to estimate the following export supply function for commodity Z:

$$Y_t = b_0 + b_1 X_t + u_t$$

Given the following information for 10 years:

$$\bar{X} = 5, \bar{Y} = 6, \sum X_t Y_t = 353, \sum X_t^2 = 304, \sum Y_t^2 = 428,$$

where, Y_t = Quantity supplied for exports of commodity Z (mil. tons)

X_t = Export price (\$ per ton)

- Obtain the OLS estimates of b_0 and b_1 .
- Compute R^2 . Give an interpretation for the coefficients of determination.
- Test the hypothesis that the quantity supplied and the price is positively related (Take $\alpha=0.05$).
- Test if export supply function should have an intercept term (Take $\alpha=0.05$).

PROBLEM 3

Consider the following demand function for chicken (1960-1982):

$$\ln Y_t = \beta_0 + \beta_1 \ln X_{t1} + \beta_2 \ln X_{t2} + \beta_3 \ln X_{t3} + \beta_4 \ln X_{t4} + u_t$$

where Y = per capita consumption of chicken, X_1 = real disposable per capita income, X_2 = real retail price of chicken, X_3 = real retail price of pork, and X_4 = real retail price of beef.

You are given the following regression results.

$$\begin{aligned} \text{(Model 1)} \quad \widehat{\ln Y}_t &= 2.1898 + 0.3425 X_{t1} - 0.5046 X_{t2} + 0.1485 X_{t3} + 0.0911 X_{t4} \\ &\quad \begin{matrix} (0.1557) & (0.0833) & (0.1109) & (0.0997) & (0.1007) \end{matrix} \\ &\quad \text{SSR}=0.013703, R^2=0.98231 \end{aligned}$$

$$\begin{aligned} \text{(Model 2)} \quad \widehat{\ln Y}_t &= 2.0328 + 0.4515 X_{t1} - 0.3772 X_{t2} \\ &\quad \begin{matrix} (0.1162) & (0.0247) & (0.0635) \end{matrix} \\ &\quad \text{SSR}= 0.015437, R^2=0.98007 \end{aligned}$$

$$\begin{aligned} \text{(Model 3)} \quad \widehat{\ln Y}_t &= 1.4910 + 0.18260 X_{t3} - 0.28695 X_{t4} \\ &\quad \begin{matrix} (0.1798) & (0.1260) & (0.1260) \end{matrix} \\ &\quad \text{SSR}= 0.089951, R^2=0.88390 \end{aligned}$$

$$\begin{aligned} \text{(Model 4)} \quad \widehat{\ln Y}_t &= 3.6639 \\ &\quad (0.0391) \\ &\quad \text{SSR}= 0.77475, R^2=? \end{aligned}$$

- Test the significance of each terms individually in Model (1) (Take $\alpha=0.05$).
- Test joint significance of the slope terms in Model (1) (Take $\alpha=0.05$).
- What do the coefficients of X_1 , X_2 , X_3 and X_4 (i.e., β_1 , β_2 , β_3 , and β_4) denote in economic theory?
- What is your theoretical expectation about the sign of β_1 ? Test your *claim* at 0.05 level of significance using Model (1).
- Suppose someone *claims* that chicken and beef are *complementary* products. Test this claim at 0.05 level of significance using Model (1).
- Test the hypothesis that $H_0: \beta_3 = \beta_4 = 0$ and interpret your results. (Take $\alpha=0.05$).
- What would be the R^2 of Model (4)?