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# Problem SET: 7

## Prob. 1

$Y = \beta_0 + \beta_1 X + U$

$\Sigma Y = 580$ $\quad \Sigma xy = -654$

$\Sigma X = 50$ $\quad \Sigma x^2 = 60$

$\Sigma XY = 2246$ $\quad \Sigma y^2 = 7568$

$\Sigma Y^2 = 41208$

$\Sigma X^2 = 310$

(a) Least square Estimates of $\beta_0$ & $\beta_1$

$$\hat{\beta}_1 = \frac{\Sigma xy}{\Sigma x^2} = \frac{-654}{60} = -10.9$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 . \bar{X} = \frac{580}{10} - (-10.9) . \frac{50}{10} = 112.5$$

(b) $$R^2 = r_{xy}^2 = r_{yx}^2 = \frac{(\Sigma xy)^2}{\Sigma x^2 \Sigma y^2} = \frac{(-654)^2}{(7568)(60)} = 0.9419$$

→ 94 percent of the variation in Y around $\bar{Y}$ is explained by X. The remaining %6 of total variation in Y is accounted for regression line & is attributed to factors included in disturbance term

(c) $H_0: \beta_0 = 0$

$H_A: \beta_0 \neq 0$

intercept term

$$t_{\hat{\beta}_0} = \frac{\hat{\beta}_0}{se(\hat{\beta}_0)} \rightarrow se(\hat{\beta}_0) = \sqrt{\frac{\sigma^2 \Sigma X^2}{T \Sigma x^2}}$$

$\sigma^2$ is pop. variance & unknown

use estimated variance $= \hat{\sigma}^2 = \dfrac{\Sigma \hat{U}_t^2}{T-k-1}$

#of exp.var.

$$R^2 = 1 - \frac{\Sigma \hat{U}_t^2}{\Sigma y^2} \Rightarrow 0.942 = 1 - \frac{\Sigma \hat{U}_t^2}{7568} \Rightarrow \Sigma \hat{U}_t^2 = 438.944$$

$$\hat{\sigma}^2 = \frac{438,944}{8} = 54.868 \qquad se(\hat{\beta}_0) = \sqrt{\frac{54.868(310)}{10(60)}} = 5.324 = se(\hat{\beta}_0)$$

$$t_{\hat{\beta}_0} = \frac{112.5}{5.324} = 21.131 > 2.306 = t^{0.025, 8} = t^{\alpha/2, T-k-1}$$

* Model Have on intercept term

$\Rightarrow$ Reject $H_0$!

* $\beta_0$ is significant at 0.05 level of significance.

(d) $H_0: \beta_1 = 0$

$H_A: \beta_1 \neq 0$

$$t_{\hat{\beta}_1} = \frac{\hat{\beta}_1}{Se(\hat{\beta}_1)} \Rightarrow Se(\hat{\beta}_1) : \sqrt{\frac{\sigma^2}{\Sigma x^2}} = \sqrt{\frac{54.868}{60}} = 0.956$$

$$t_{\hat{\beta}_1} = \frac{-10.9}{0.956} = |-11.402| > 2.306 \Rightarrow$$ Reject $H_0$! $\hat{\beta}_1$ is significant at 0.05 sign. level.

(e) CI for %99

$$\hat{\beta}_1 \pm se(\hat{\beta}_1) . \underbrace{t^{\alpha/2, T-k-1}}_{3.355} \Rightarrow -10.9 \pm (0.956). 3.355$$

$[-7.693, 14.107]$

## Problem 2

$Y_t = b_0 + b_1 X_t + U_t$  $X_t$ = Export Price

$\bar{X} = 5$ ; $\bar{Y} = 6$, $n = 10$  $Y_t = Q^S$ for exports

$\sum X_t Y_t = 353$

$\sum X_t^2 = 304$

$\sum Y_t^2 = 428$

(a) $b_0 = \bar{Y} - \hat{\beta}_1 . \bar{X} \Rightarrow 6 - (0.981) \times 5 = 1.093$

$$b_1 = \frac{\sum X_t Y_t - T.\bar{X}.\bar{Y}}{\sum X_t^2 - T.\bar{X}^2} = \frac{353 - (10)(5)(6)}{304 - (10.)5^2} = \frac{53}{54} = 0.981$$

(b) $R^2 = r_{xy}^2 \Rightarrow 0.76498$

$$r_{yx} = \frac{T \sum X.Y - \sum X \sum Y}{\sqrt{[T\sum X^2 - (\sum X)^2]\,[T\sum Y^2 - (\sum Y)^2]}} = \frac{10(353) - (10.5)(10.6)}{(10(304) - 50^2)(10(428) - 60^2)} = \frac{530}{605.97} = 0.8746$$

$\Rightarrow$ About 87 percent of variation in Y around its mean is explained by X.

(c) $H_0: b_1 = 0$

$H_A: b_1 > 0$

$$t_{\hat{\beta}_1} = \frac{\hat{\beta}_1}{Se(\hat{b}_1)} = \frac{0.981}{0.140} = 7.007 \qquad t^{\alpha, T-k-1} = t^{0.05, 8} = 1.860$$

$$R^2 = 1 - \frac{\sum \hat{U}_t^2}{\sum y^2} \Rightarrow \sum \hat{U}_t^2 = 8.527 \qquad \sigma^2 = \frac{\sum \hat{U}_t^2}{T-k-1} = 1.0659 \qquad Se(\hat{b}_1) = \sqrt{\frac{\sigma^2}{\sum x^2}} = \sqrt{\frac{1.0659}{54}} = 0.140$$

$7.007 > 1.860 \Rightarrow$ Reject $H_0$ ∇ → $b_1$ is positive. There is a positive relation btw X & Y ∇

→ Data support the existence of this positive relationship.

(d) $H_0: b_0 = 0$

$H_A: b_0 \neq 0$

$$t_{b_0} = \frac{\hat{\beta}_0}{Se(\beta_0)} \qquad Se(\beta_0) = 0.7746 \qquad \sigma^2 = 1.0659$$

$$t_{b_0} = \frac{1.093}{0.7746} = 1.4109 \qquad t^{0.025, 8} = 2.306 \qquad \text{DNR } H_0 \nabla$$

So, Intercept term is not significant.

Hence, the export supply function should not have an intercept term. The existance of the intercept term in the regression model does not supported by data.

**Prob. 3** $\ln y_t = \beta_0 + \beta_1 \ln X_{t1} + \beta_2 \ln X_{t2} + \beta_3 \ln X_{t3} + \beta_4 \ln X_{t4} + u_t$ (1960 - 1982)

Model 1

(a) $H_0: \beta_0 = 0$, $H_A: \beta_0 \neq 0$ $\quad t_{\hat\beta_0} = \dfrac{2.1898}{0.1557} = 14.064$ R. $H_0$!

$\Rightarrow H_0: \beta_1 = 0$, $H_A: \beta_1 \neq 0$ $\quad t_{\hat\beta_1} = \dfrac{0.3425}{0.0833} = 4.112$ R. $H_0$!

$\Rightarrow H_0: \beta_2 = 0$, $H_A: \beta_2 \neq 0$ $\quad t_{\hat\beta_2} = \dfrac{-0.5046}{0.1109} = -4.55$ R. $H_0$!

$\Rightarrow H_0: \beta_3 = 0$, $H_A: \beta_3 \neq 0$ $\quad t_{\hat\beta_3} = \dfrac{0.1485}{0.0997} = 1.489$ DNR $H_0$!

$\Rightarrow H_0: \beta_4 = 0$, $H_A: \beta_4 \neq 0$ $\quad t_{\hat\beta_4} = \dfrac{0.0911}{0.1007} = 0.9047$ DNR $H_0$!

$$\boxed{t^{\alpha/2,\, T-k-1} = t^{0.025,\, 18} = 2.101}$$

* k: exp. variable
* T: # of obs.

* $\beta_0, \beta_1, \beta_2$ significant, $\beta_3, \beta_4$ insignificant

(b) Joint significant Test

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

$H_A$: at least one of them is non-zero

$$\theta = \frac{(SSR_R - SSR_U)/p}{SSR_U / T-k-1} \sim F^{\alpha}_{p,\, T-k-1}$$

* p: # of restriction
* k: exp. variable

* Use the $\theta$ statistic calculated using SSR's.
* Do not use " " " " $R^2$.
since it is not done in lectures.

$\rightarrow y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \beta_3 X_{t3} + \beta_4 X_{t4} + u_t \Rightarrow$ unrestricted model: $SSR_U$

$\rightarrow \hat y_t = \hat\beta_0 \Rightarrow$ restricted model: $SSR_R$.

Do not use $\theta$ statistic calculated using $R^2$ !!!!

in question: Model 1 unrest., model 4 restricted!

$$\theta = \frac{(0.77475 - 0.013704)/4}{(0.013704)/18} = 249.92$$

$F^{\alpha}_{p,\, T-k-1} = F^{0.05}_{4,18} = 2.93$

$\theta > F^{\alpha} \Rightarrow$ Reject $H_0$!

(c) Modelin tümü ln'li ise $\beta_1, \underbrace{\beta_2, \beta_3, \beta_4}_{\text{price elast}}$ birer elasticity verir. (income elast: $\beta_1$)

$\beta_1 \rightarrow$ income elasticity of chicken demand
$\beta_2 \rightarrow$ own price elasticity of chicken demand
$\beta_3 \rightarrow$ cross-price elasticity of chicken demand with respect to pork
$\beta_4 \rightarrow$ cross-price elasticity of chicken demand wrt. beef.

(d) $H_0: \beta_1 \leq 0$, $H_A: \beta_1 > 0$ $\quad t_{\hat\beta_1} = \dfrac{0.3425}{0.0833} = 4.116$ $\quad t^{\alpha,\, T-k-1} = t^{0.05,\, 18} = 1.734$

$4.116 > 1.734 \Rightarrow$ Reject $H_0$! positively related

(e) $\beta_4 > 0$ chicken and beef are substitutes
$\beta_4 < 0$ chicken and beef are complements
$\beta_4 = 0$ unrelated goods

$H_0: \beta_4 \geq 0$, $H_A: \beta_4 < 0$ $\quad t_{\hat\beta_4} = \dfrac{0.0911}{0.1007} = 0.9047$

$t^{\alpha}_{T-k-1} = 1.734 \Rightarrow$ Do not R$H_0$! $\Rightarrow$ So they are not complementary goods.

(f) $H_0: \beta_3 = \beta_4 = 0$

$H_A: \beta_3 \neq \beta_4 \neq 0$

$\alpha = 0.05$

$\underbrace{\phantom{xxxxxxx}}$
Soru setinde yanlış verilmiş test.

Model 1 : unrest.

Model 2 = restricted.

$$\theta = \frac{(SSR_R - SSR_U)/P}{SSR_U/(T-k-1)} = \frac{(0.015437 - 0.013703)/2}{(0.013703)/18} \cong 1.1389$$

$F^{0.05}_{2,18} = 3.55 < 1.1389 \Rightarrow$ DNR Ho!

$\beta_3$ and $\beta_4$ are not jointly significant.

(g) $R^2 = 0 \Longrightarrow R^2 = \dfrac{\hat{\beta}_1 \sum x_t^2}{\sum y_t^2} \Rightarrow$


$R^2$, Sağdaki x'lerin bağımlı değişkenin ortalamasından sapmasının ne kadarını açıkladığını verir. Sağda x olmadığı zaman $R^2 = 0$ olacaktır.

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Never Forget: Standard deviation or standard error may <u>never</u> be negative!