Middle East Technical University **2010-2011 Fall**
Department of Economics
ECON 206

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PROBLEM SET 01
EXPECTATION AND VARIANCE OPERATORS

PROBLEM 1 *(Exercise 4.17 from textbook)*

Prove Theorem 4.5.

Theorem 4.5: Let c be a constant, and let $g(Y), g_1(Y), g_2(Y), \ldots, g_k(Y)$ be functions of a continuous random variable *Y.* Then the following results hold:

1. E(c) = c.
2. E[cg(Y)] = cE[g(Y)].
3. $E[g_1(Y)+ g_2(Y)+ \ldots +g_k(Y)] = E[g_1(Y)]+ E[g_2(Y)]+ \ldots +E[g_k(Y)]$.

PROBLEM 2 *(Exercise 4.18 from textbook)*

If *Y* is a continuous random variable with density function *f(Y),* use Theorem 4.5 to prove that $\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2$.

PROBLEM 3 *(Exercise 4.20 from textbook)*

If *Y* is a continuous random variable with mean μ and variance σ^2 and *a* and *b* are constants, use Theorem 4.5 to prove the following:

a. $E(aY+b)=aE(Y)+b=a\mu+b$.
b. $V(aY+b) = a^2V(Y) = a^2\sigma^2$

PROBLEM 4 *(Exercise 4.24 from textbook)*

The proportion of time *Y* that an industrial robot is in operation during a 40-hour week is a random variable with probability density function

$$f(y) = \begin{cases} 2y & , \quad 0 \le y \le 1 \\ 0 & , \quad elsewhere \end{cases}$$

a. Find *E(Y)* and *V(Y).*

b. For the robot under study, the profit X for a week is given by $X = 200Y - 60$. Find $E(X)$ and $V(X)$.

c. Find an interval in which the profit should lie for *at least* 75% of the weeks that the robot is in use.

PROBLEM 5

Let $X \sim N(\mu, \sigma^2)$. Use the algebra of expectations and find $Var(\bar{X})$.

PROBLEM 6

Let $X \sim N(\mu, \sigma^2)$. Use the algebra of expectations and find $E(\bar{X}^2)$.

PROBLEM 7

Let $Z = \dfrac{X - \mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$. Use the algebra of expectations and find $E(Z)$ and $V(Z)$.

PROBLEM 8

Using the properties of the expectation operator, show that;

a. $Var(aX) = a^2 Var(X)$

b. $Var\left(a + bX\right) = b^2 Var(X)$

PROBLEM 9

If X and Y are two random variables and a and b are constants, prove that;

a. $Var\left(X + a\right) = Var\left(X\right)$

b. $Var\left(Y + b\right) = Var\left(Y\right)$

PROBLEM 10

X_1, X_2, X_3 are independent random variables such that $E(X_i) = 0$ and $E(X_i^2) = 1$. Calculate $E[X_1^2\left(X_2 - 4X_3\right)^2]$.

PROBLEM 11 *(Exercise 7.7, excluding c, from textbook)*

Suppose that X_1, X_2... , X_m and Y_1, Y_2, ... Y_n are independent random samples with the variables X_i normally distributed with mean μ_1 and variance σ_1^2 and the variables Y_i normally distributed with mean μ_2 and variance σ_2^2. The difference between the sample means, $\bar{X} - \bar{Y}$, is then a linear combination of *m+n normally* distributed random variables and, by Theorem 6.3, is itself normally distributed.

a. Find E($\bar{X} - \bar{Y}$).

b. Find V ($\bar{X} - \bar{Y}$).