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PROBLEM SET 04

PROBLEM 1

In a game, a participant is given three attempts to hit a ball. On each try, she either scores a hit, H or a miss, M . The game requires that the player must alternate which hand she uses in successive attempts. That is, if she makes her first attempt with her right hand, she must use her left hand for the second attempt and her right hand for the third. Her chance of scoring a hit with her right hand is 0.7 and with her left hand is 0.4. Assume that the results of successive attempts are independent and that she wins the game if she scores at least two hits in a row. If she makes her first attempt with her right hand what is the probability that she wins the game?

PROBLEM 2

Two events A and B are such that $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$. Find the following:

- $P(A \cap B)$
- $P(\bar{A} \cup \bar{B})$
- $P(\bar{A} \cap \bar{B})$
- $P(\bar{A} | B)$

PROBLEM 3

If A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.2$, find the following:

- $P(A \cup B)$
- $P(\bar{A} \cap \bar{B})$
- $P(\bar{A} \cup \bar{B})$

PROBLEM 4

Diseases I and II are prevalent among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease eventually and 3% will contract both diseases.

- a. Find the probability that a randomly chosen person from this population will contract at least one disease.
- b. Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.

PROBLEM 5

A population of voters contains 40% Republicans and 60% Democrats. It is reported that 30% of the Republicans and 70% of the Democrats favor an election issue. A person chosen at random from this population is found to favor the issue in question. Find the conditional probability that this person is a Democrat.

PROBLEM 6

A diagnostic test for a disease is said to be 90% accurate in that if a person has the disease, the test will detect it with probability 0.9. Also, if a person does not have the disease, the test will report that he or she does not have it with probability 0.9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease, what is the conditional probability that she does, in fact, have the disease? Are you surprised by the answer? Would you call this diagnostic test reliable?

PROBLEM 7

Medical case histories indicate that different illnesses may produce identical symptoms. Suppose that a particular case of symptoms, denoted H , occurs only when one of three illnesses, I_1 , I_2 and I_3 , occurs. Assume that the simultaneous occurrence of more than one of these illnesses is impossible and that

$$P(I_1) = 0.01, P(I_2) = 0.005, P(I_3) = 0.02$$

The probabilities of developing the set symptoms H , given each of these illnesses are known to be

$$P(H|I_1) = 0.90, P(H|I_2) = 0.95, P(H|I_3) = 0.75$$

Assuming that an ill person exhibits the symptoms, H , what is the probability that the person has illness I_1 ?

PROBLEM 8

A plane is missing and is presumed to have equal probability of going down in any of three regions. If a plane is actually down in region i , let $1 - \alpha_i$ denote the probability that the plane will be found upon a search of the i th region, $i=1, 2, 3$.

- What is the conditional probability that the plane is in region 1 given that the search of region 1 was unsuccessful?
- What is the conditional probability that the plane is in region 2 given that the search of region 1 was unsuccessful?
- What is the conditional probability that the plane is in region 3 given that the search of region 1 was unsuccessful?

PROBLEM 9

Five identical bowls are labeled 1,2,3,4, and 5. Bowl i contains i white and $5-i$ black balls with $i=1,2,\dots,5$. A bowl is randomly selected and two balls are randomly selected (without replacement) from the contents of the bowl.

- What is the probability that both balls selected are white?
- Given that both balls selected are white, what is the probability that bowl 3 was selected?

PROBLEM 10

A spinner can land in any of the four positions A, B, C and D, with equal probability. The spinner is used twice and the position is noted each time. Let the random variable Y denote the number of positions on which the spinner did *not* land. Compute the probabilities for each value of Y .