

# PROBLEM SET-MODEL SELECTION

a) The logic behind constructing a General Unrestricted Model (GUM) is to start the model selection process with the most comprehensive model possible so that a general to specific procedure can be followed. This means that we will apply restrictions at each step to obtain a more precise model, until we reach the one that is parsimonious (the simplest model that is most explanatory). This procedure requires us to carry out diagnostic tests at each step and abandon the simplification path if the model fails in any of these tests (see handout 15). Therefore, we require GUM to be the most comprehensive model possible while not failing in any of the mis-specification tests.

First, among the equations that we have, note that model 1 is the most comprehensive one in the sense that models 2 and 3 are obtained by applying restrictions on it (Equations 4 and 5 are estimated in order to carry out the tests of part (b)). This makes Equation 1 a candidate for GUM. We then need to carry out diagnostic tests.

(Note that we reject  $H_0$  if,  $p < \alpha$  (significance level))

(Zaremska test is used to determine the functional form, but it is not given in the question)  
 Jarque-Bera test = Under  $H_0$ , the sample data resembles the normal distribution of disturbances, which we do not reject since  $p = 0.58 > 0.05$ .

RESET test = Under  $H_0$ , there is no specification error caused by ignoring (quadratic) non-linearities, DNR since  $p = 0.37 > 0.05$ .

CHOW tests =  $CHOW_{\text{in-sample}}$  is the test for in-sample stability with  $H_0$  being no structural change. Likewise  $CHOW_{\text{predictive}}$  is for out-of-sample stability. Both were not rejected as  $p = 0.59$  and  $p = 0.64$ .

CUSUM = For CUSUM and CUSUM<sub>sc</sub> we can look at the given plots. Since the plots do not cross the blue lines, these tools do not suggest structural change.

Durbin-Watson (DW) test is carried out to test for autocorrelation, white is for heteroscedasticity and ARCH is for ARCH effects. All these tests imply that there is no autocorrelation, heteroscedasticity or ARCH effects present. Additionally we have considerably high  $R^2$ .

Eq 1, then, is the most comprehensive model that does not fail in any of these diagnostic tests. Therefore it can be adopted as GUM.

b) In the sequential testing process one should always test the restrictions used to obtain the restricted models. Here, since M2 and M3 are selected as a result of a sequential testing procedure, the restrictions used to obtain M2 and M3 hold. Also note that in Eq 2 we have an insignificant coefficient. GUM ends where there are no more insignificant coefficients or omitting an insignificant coefficient would lead the simpler model to fail some diagnostic tests. Here since we do not have the simpler model and we are asked to choose between Eq 2 and Eq 3, this means these two models are the final two models that we need to choose from without further applying restrictions.

As a comparison criterion note that we cannot use  $R^2$  as the number of explanatory variables are different in each equation, and the same goes for SSR. Instead we can use adjusted  $R^2$  and AIC or SIC. Eq 2 is better in terms of these statistics ( $\bar{R}^2$  is higher and AIC/SIC are lower), however we still need to carry out additional tests before concluding. (Also note that both models pass the diagnostic tests described in part (a).)

Using equations (4) and (5), we can carry out Davidson-Mackinnon J test. This test is carried out by taking the fitted values from one model, adding them as an explanatory variable to the other and then testing the significance of its coefficient. If it is significant, then the fitted values have additional explanatory power and vice versa.

$$\text{Eq 4: } \ln \hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 (\ln X_t - \ln W_t) + \hat{\beta}_2 (\ln Z_t - \ln W_t) + \hat{\beta}_3 \ln \hat{Y}_t \quad \text{Eq 3}$$

$$\left. \begin{array}{l} H_0: \beta_3 = 0 \\ H_A: \beta_3 \neq 0 \end{array} \right\} p = 0.49 \rightarrow \text{DNR } H_0 \text{ under } 0.05 \text{ significance level.}$$

(or compare  $t=0.71$  with  $t_{(6)}$ )

$$\text{Eq 5: } \ln \hat{Y}_t = \hat{\alpha}_0 + \hat{\alpha}_1 \ln Z_t + \hat{\alpha}_2 \ln W_t + \hat{\alpha}_3 \ln \hat{Y}_t \quad \text{Eq 2}$$

$$\left. \begin{array}{l} H_0: \alpha_3 = 0 \\ H_A: \alpha_3 \neq 0 \end{array} \right\} p = 0.07 \rightarrow \text{DNR } H_0 \text{ under } 0.05 \text{ significance level.}$$

(or  $t=1.91$ )

When we do not reject the null hypothesis for both equations' fitted values, this means that they have no additional explanatory power over one another. (As an example, if we had 10% significance level, then we would reject  $H_0$ , and choose Eq 2).

When we do not reject the null in both of these tests, we say that both of the models are accepted according to the Davidson-Mackinnon J test.

As a conclusion, even though Eq 3 is simpler and both models can be accepted, Eq 2 has better statistics as discussed before. Therefore we can choose Eq 2.