PROBLEM SET 3

Problem 1

(I) $\ln M_t^n = \beta_0 + \beta_1 \ln Y_t^A + \beta_2 \ln r_t + \beta_3 \ln P_t + u_t$

a) we should first look at whole significance:

- $H_0 = \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$
  $H_A =$ at least one of them differs

$$Q = \frac{R^2}{1-R^2} \cdot \frac{T-k-1}{p} = \frac{0.9418}{1-0.9418} \cdot \frac{17-3-1}{4} \cong 52,59$$

$52,59 > F_{4,13}^{0,05} = 3,17 \Rightarrow RH_0$, the model is wholly significant.

- Individual significance tests:

i)	ii)	iii)	iv)
$H_0 = \beta_0 = 0$	$H_0 = \beta_1 = 0$	$H_0 : \beta_2 = 0$	$H_0 : \beta_3 = 0$
$H_A : \beta_0 \neq 0$	$H_A = \beta_1 \neq 0$	$H_A : \beta_2 \neq 0$	$H_A : \beta_3 \neq 0$
$t_{calc} = \frac{3.9995}{1.8013} = 2,22$	$\frac{1,7106}{0,4155}$ 4,12	$\frac{-0,6079}{0,4687}$ $-1,3$	$\frac{-0,7587}{0,6505}$ $-1,17$

with $t_{0.025,13} = 2,16$;

$\lvert 2.22 \rvert > 2,16$	$\lvert 4,12 \rvert > 2,16$	$\lvert -1,3 \rvert < 2,16$	$\lvert -1,17 \rvert < 2,16$
RH_0	RH_0	DNR H_0	DNR H_0
β_0 is individually significant	β_1 is individually significant	β_2 is individually insignificant	β_3 is individually insignificant

- We have high R^2, whole significance with 2 out of 3 independent variables' coefficients being individually insignificant. Also, the sign of β_3 is theoretically inconsistent. All these findings may indicate the existence of serious multicollinearity.

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In order to see which variable contributes the most to this problem, we can check VIF's:

For $\ln Y_t \Rightarrow \dfrac{1}{1-\rho_1^2} = \dfrac{1}{1-0.9625} \cong 26.7$

$\ln r_t \Rightarrow \dfrac{1}{1-\rho_2^2} = \dfrac{1}{1-.9078} = 10.84$

$\ln P_t \Rightarrow \dfrac{1}{1-\rho_3^2} = \dfrac{1}{1-0.8789} = 8.25$

$\ln Y_t$ is the variable that contributes the most to this problem since VIF is highest for $\ln Y_t$.

b) – Theoretically there should be a positive relationship between Y and money demand. As income increases, the demand for holding money should also increase. Omitting this variable would be ignoring this relationship as if income has no effect on money demand, therefore economically it is not advisable.

– There is a high collinearity between Y_t and other explanatory variables, which indicates that there will be considerable correlation with other independent variables, or with their linear combinations.

Suppose we had $Z_t = \alpha_1 + \alpha_2 X_{1t} + \alpha_3 X_{2t} + U_t$ and we drop X_{2t} due to collinearity. We will then have $Z_t = \beta_0 + \beta_1 X_{1t} + \varepsilon_t$

$$\hat{\beta}_1 = \frac{\sum x_{1t} z_t}{\sum x_{1t}^2} = \frac{\sum x_{1t}(Z_t - \bar{Z})}{\sum x_{1t}^2} = \frac{\sum x_{1t} Z_t}{\sum x_{1t}^2} - \underbrace{\frac{\bar{Z}\sum x_{1t}}{\sum x_{1t}^2}}_{=0 \text{ since } \sum x_{1t} = 0}$$

$$= \frac{\sum x_{1t}(\beta_0 + \beta_1 X_{1t} + \varepsilon_t)}{\sum x_{1t}^2} = \underbrace{\frac{\beta_0 \sum x_{1t}}{\sum x_{1t}^2}}_{=0\ (\sum x_{1t}=0)} + \beta_1 \underbrace{\frac{\sum X_{1t} x_{1t}}{\sum x_{1t}^2}}_{=1} + \frac{\sum x_{1t}\varepsilon_t}{\sum x_{1t}^2}$$

(↳ found in earlier chapters, you can verify this by looking at $\dfrac{\sum X_{1t}(X_{1t}-\bar{X})}{\sum x_{1t}^2}$)

Therefore we have:

$$\hat{\beta}_1 = \beta_1 + \frac{\sum x_{1t}\varepsilon_t}{\sum x_{1t}^2}$$

$(\mathrm{Cov}(X_{1t}, \varepsilon_t) \neq 0)$

Now ε_t will include X_{2t}, which had considerable correlation with X_{1t}. Therefore $\sum x_{1t}\varepsilon_t \neq 0$ and also $E(x_{1t}\varepsilon_t) \neq 0$. The higher the correlation, the more biased will be $\hat{\beta}_1$.

(If X is fixed, $E(X_{1t}\varepsilon_t) = X_{1t} E(\varepsilon_t)$ the resulting condition becomes $E(\varepsilon_t) \neq 0$)

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③ $E(\hat{\beta}_1) \neq \beta_1$ as $E(\hat{\beta}_1) = \beta_1 + \frac{\sum x_t E(\varepsilon_t)}{\sum x_t^2}$ and $E(\varepsilon_t) \neq 0$

In regression (2), omitting Y makes both explanatory variables' coefficients individually significant:

$$\ln r_t \Rightarrow t_c = \frac{-1{,}1571}{0{,}2767} = |-4{,}67| > t_{crit} \quad RH_0$$

$$\ln P_t \Rightarrow t_c = \frac{1{,}6555}{0{,}4399} = 3{,}76 > t_{crit} \quad RH_0$$

$(t_{crit} = t_{0{,}025,14} = 2{,}14)$

This is due to a tradeoff between variance and biasedness. Omitting a variable causes the estimators to be biased but also have smaller variances.

2) i) $Q_t = \beta_0 L_t^{\beta_1} e^{u_t}$

ii) $Y_t = \beta_0 \beta_1^{X_t} e^{u_t}$

iii) $e^{Y_t} = \beta_0 X_t^{\beta_1} e^{u_t}$

iv) $\frac{1}{Y_t} = \beta_0 + \beta_1 \frac{1}{X_t} + u_t$

i) $\ln Q_t = \underbrace{\ln\beta_0}_{\beta_0^*} + \beta_1 \ln L_t + u_t$

$\ln Q_t = \beta_0^* + \beta_1 \ln L_t + u_t \Rightarrow$ Log-linear, we can apply OLS

ii) $\ln Y_t = \underbrace{\ln\beta_0}_{\beta_0^*} + \underbrace{\ln\beta_1}_{\beta_1^*} X_t + u_t$

$\ln Y_t = \beta_0^* + \beta_1^* X_t + u_t \Rightarrow$ Log-lin, we can apply OLS.

iii) $Y_t = \underbrace{\ln\beta_0}_{\beta_0^*} + \beta_1 \ln X_t + u_t$

$Y_t = \beta_0^* + \beta_1 \ln X_t + u_t \Rightarrow$ Lin-log, we can apply OLS.

iv) $\frac{1}{\underbrace{Y_t}_{Y_t^*}} = \beta_0 + \beta_1 \frac{1}{\underbrace{X_t}_{X_t^*}} + u_t$

$Y_t^* = \beta_0 + \beta_1 X_t^* + u_t \Rightarrow$ Linear regression, we can apply OLS.

As $X \to \infty$, $Y \to \frac{1}{\beta_0}$

This can be used to explain inferior goods.

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3) (I) $\ln Y_t = \ln \beta_0 + \beta_1 \ln X_{t1} + u_{t1}$

$$\frac{\partial \ln Y_t}{\partial \ln X_{t1}} = \frac{\frac{\partial Y_t}{Y_t}}{\frac{\partial X_{1t}}{X_{1t}}} = \frac{\partial Y_t}{\partial X_{t1}} \frac{X_{t1}}{Y_t} = \beta_1$$

The elasticity is equal to β_1, which is the relative change in Y with respect to a relative change in X_1. If X_1 increases by 1%, Y also increases by β_1 % (or if β_1 is negative, Y decreases as X increases). The assumption here is that the elasticity is β_1, which is constant.

(II) $Y_t = \alpha_0 + \alpha_1 X_{t1} + u_{t2}$

$$\frac{\partial Y_t}{\partial X_{t1}} = \alpha_1 \qquad \text{Elasticity} = \frac{\partial Y_t}{\partial X_{t1}} \cdot \frac{X_{t1}}{Y_t} = \alpha_1 \frac{X_{t1}}{Y_t}$$

Elasticity depends on the values of Y and X_1 here, so it is not constant. α_1 shows the unit change in Y caused by a unit change in X_1.

(III) $\ln Y_t = \gamma_0 + \gamma_1 X_{t1} + u_{t3}$

$$\gamma_1 = \frac{\partial \ln Y_t}{\partial X_{t1}} = \frac{\frac{\partial Y_t}{Y_t}}{\partial X_{t1}} = \frac{1}{Y_t} \frac{\partial Y_t}{\partial X_{t1}}$$

$$\text{Elasticity} = \frac{\partial Y_t}{\partial X_{t1}} \frac{X_{t1}}{Y_t} = \gamma_1 X_{t1}$$

Here elasticity depends on the value of X_1. γ_1 shows the relative change in Y caused by a unit change in X_1. (If we multiply γ_1 with 100, we find the percentage change in Y)

4) $Y_t = b_0 + b_1 X_{t1} + b_2 X_{t2} + b_3 X_{t3} + u_t$

$$X_{t2} + X_{t3} = 1$$

* Note that $X_{t2} = 1 - X_{t3}$ (or $X_{t3} = 1 - X_{t2}$), which means X_2 and X_3 are linear functions of each other without any other variability. This shows perfect multicollinearity.

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5

Plugging $X_{t2} = 1 - X_{t3}$ in the model:

$$Y_t = b_0 + b_1 X_{t1} + b_2 (1 - X_{t3}) + b_3 X_{t3} + u_t$$

$$= b_0 + b_1 X_{t1} + b_2 - b_2 X_{t3} + b_3 X_{t3} + u_t$$

$$= \underbrace{b_0 + b_2}_{\beta_0} + \underbrace{b_1}_{\beta_1} X_{t1} + \underbrace{(b_3 - b_2)}_{\beta_2} X_{t3} + u_t$$

$$Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t3} + u_t$$

When we apply OLS here, we obtain the values of β_0, β_1 and β_2.
However,

$$\left.\begin{aligned} \beta_0 &= b_0 + b_2 \\ \beta_1 &= b_1 \\ \beta_2 &= b_3 - b_2 \end{aligned}\right\}$$

we have 3 equations for 4 unknowns.
Therefore we cannot obtain individual regression coefficients from here.

When we apply OLS in a multiple regression, individual regression coefficients show the pure effects of X_2 and X_3 on Y. However, here there is no variation in X_2 when we account for the variation in X_3 and vice versa. As a result of this, we cannot separate the two and obtain their individual effects.

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