PROBLEM SET 4 - ANSWERS

1) Here the researcher will encounter what is called the "dummy variable trap." As $D_{1t}+D_{2t}=D_{3t}+D_{4t}=1$, there will be perfect multicollinearity among the related variables, and the researcher will not be able to obtain the estimators of these variables' coefficients. Remember that in a multiple regression model, each coefficient shows their respective variables partial, or pure, effect. In this example, D_{1t} and D_{2t} always move together so that the pure effects of these variables cannot be estimated (the same holds for D_3 and D_4 as well).

In order to overcome this problem one should use n-1 dummies for n categories. Here for male and female there should be only one dummy variable, which can be obtained by substituting $D_{1t}=1-D_{2t}$ (or $D_{2t}=1-D_{1t}$) in the model. (second parameterization), and the same procedure should also be followed for college education.

Finally, note that the dummy variable trap is not related to the number of explanatory variables but to the number of categories and the number of dummies used. In this example there are no explanatory variables, yet there will be two dummies after second parameterization and it will not cause a dummy variable trap.

(One other way to eliminate the trap is to eliminate the intercept term, but this is not much advised unless there is no other option).

2) a)(i) For 1951-1959 ($D_t=0$)

$$\hat{I}_t = 1633,55 + 154,32Y_t + 0,12M_t$$

(ii) For 1960-1976 ($D_t=1$)

$$\hat{I}_t = 1633,55 + 154,32Y_t + 0,12M_t + 584,54 - 41,42Y_t + 0,34M_t$$

$$= 2218,09 + 112,9Y_t + 0,46M_t$$

b) $\hat{I}_t = 996.5 + 116.19 Y_t + 0.37 M_t$ ($t = 1951-1976$) $SSR = 493245$

This model assumes no structural change, so it will become the restricted model, whereas the unrestricted model will be:

$$\hat{I}_t = 1633{,}55 + 154{,}32 Y_t + 0{,}12 M_t + \underbrace{584{,}54}_{\alpha_0} D_t - \underbrace{41{,}42}_{\alpha_1} D_t Y_t + \underbrace{0{,}34}_{\alpha_2} D_t M_t$$

So $SSR_R = 493245$ and $SSR_U = \sum (I_t - \hat{I}_t)^2 = 67589$ (given in the question)

H_0: $\alpha_0 = \alpha_1 = \alpha_2 = 0$ (no structural change)
H_A: at least one differs

$$Q = \frac{SSR_R - SSR_U}{SSR_U} \times \frac{T - 2(\overset{\text{number of slope terms in the restricted model}}{k}+1)}{p}$$

$$= \frac{493245 - 67589}{67589} \times \frac{26 - 2(2+1)}{3} = 41{,}98$$

$F_{crit} = F^{0.05}_{3,20} = 3{,}10$ $\quad Q > F_{crit} \Rightarrow RH_0$.

There is a structural change in the investment function.

c) First parameterization → define 2 dummies for 2 categories

$$D_{t1} = \begin{cases} 1 & \text{if } t = 1951-1959 \\ 0 & \text{if } t = 1960-1976 \end{cases}$$

$$D_{t2} = \begin{cases} 0 & \text{if } t = 1951-1959 \\ 1 & \text{if } t = 1960-1976 \end{cases}$$

1st parameterization (combining both subsamples from part (a) with dummies):

$$\Rightarrow \hat{I}_t = D_{t1}\, 1633{,}55 + D_{t1}\, 154{,}32 Y_t + D_{t1}\, 0{,}12 M_t + D_{t2}\, 2218{,}09 + D_{t2}\, 112{,}9 Y_t + D_{t2}\, 0{,}46 M_t$$

2^{nd} parameterization - substitute $D_{t1} = 1 - D_{t2}$ (this means we take the first subsample as the base. You may as well take $D_{t2} = 1 - D_{t1}$)

$$\Rightarrow \hat{I}_t = 1633{,}55 - 1633{,}55 D_{t2} + 154{,}32 Y_t - 154{,}32 D_{t2} Y_t + 0{,}12 M_t - 0{,}12 D_{t2} M_t + D_{t2}\, 2218{,}09 + D_{t2}\, 112{,}9 Y_t + D_{t2}\, 0{,}46 M_t$$

$$\Rightarrow \hat{I}_t = 1633{,}55 + 154{,}32 Y_t + 0{,}12 M_t + D_{2t}\, 584{,}54 - D_{t2}\, 41{,}42 Y_t + D_{t2}\, 0{,}34 M_t$$

Note that this is the combined investment function given at the beginning of the question. In part (a) we separated it into two subsamples. In this part we combined the two subsamples back.

Problem 3

a) Define $D_{1t} = \begin{cases} 1 & \text{if } t = 1946\text{-}1960 \\ 0 & \text{if } t = 1961\text{-}1980 \end{cases}$, $D_{2t} = \begin{cases} 0 & \text{if } t = 1946\text{-}1960 \\ 1 & \text{if } t = 1961\text{-}1980 \end{cases}$

1^{st} parameterization:

$$\hat{Y}_t = 1{,}67 D_{1t} L_t + 2{,}58 D_{1t} K_t + 1{,}89 D_{2t} L_t + 3{,}12 D_{2t} K_t$$

2^{nd} parameterization: $(D_{1t} = 1 - D_{2t})$

$$\hat{Y}_t = 1{,}67 (1 - D_{2t}) L_t + 2{,}58 (1 - D_{2t}) K_t + 1{,}89 D_{2t} L_t + 3{,}12 D_{2t} K_t$$

$$\circledast \quad \hat{Y}_t = \underbrace{1{,}67}_{\hat{\beta}_1} L_t + \underbrace{2{,}58}_{\hat{\beta}_2} K_t + \underbrace{0{,}22}_{\hat{\beta}_3} D_{2t} L_t + \underbrace{0{,}54}_{\hat{\beta}_4} D_{2t} K_t$$

This is a combined production function which allows marginal productivities of labor and capital to differ between subperiods.

b) $\hat{\beta}_1 \to$ 1 unit Δ in L_t causes a 1,67 unit Δ in Y_t if $t \in$ 1946-1960
$\hat{\beta}_2 \to$ 1 unit Δ in K_t causes a 2,58 unit Δ in Y_t if $t \in$ 1946-1960

$\hat{\beta}_3 \to$ When $t \in$ 1961-1980, the marginal "effect (productivity) of L_t increases by 0,22 with respect to t= 1946-1960. 1 unit Δ in L_t causes a 1,89 unit Δ in Y_t if $t \in$ 1961-1980.

$\hat{\beta}_4 \to$ For $t \in$ 1961-1980, the marginal effect of K_t increases by 0,54 w.r.t. 1946-1960, so 1 unit Δ in K_t causes a 3,12 Δ in Y_t if $t \in$ 1961-1980.

c) Here we will use Chow's (ANCOVA) test:
SSR of the unrestricted model is

$$SSR_U = \underbrace{SSR_1}_{\text{1st subsample}} + \underbrace{SSR_2}_{\text{2nd subsample}} = 0{,}2116 + 0{,}3600 = 0{,}5716$$

Restricted model is the one that implies no structural change, model 3.
Therefore $SSR_R = 1{,}6216$

$H_0: \beta_3 = \beta_4 = 0$
H_A: o.w.

Since there is no intercept term, $k+1$ becomes k, and $k=2$ so $2(k+1) \to 4$

$$Q = \frac{1{,}6216 - 0{,}5716}{0{,}5716} \times \frac{35-4}{2} = 28{,}53 > F^{0{,}05}_{2,31} = 3{,}32$$

$RH_0 \to$ there is a structural change between the subperiods

d) Since the question asks "and/or" we need individual test for the "or" part and joint significance for the "and" part. We carried out the test for the joint case above, so we will look at individual significances here.

Our combined production function was:

$$\hat{Y}_t = \hat{\beta}_1 L_t + \hat{\beta}_2 K_t + \hat{\beta}_3 D_{2t} L_t + \hat{\beta}_4 D_{2t} K_t$$

If we write models (1) and (2) in terms of parameters (instead of parameter values)

(1) $\hat{Y}_t = \hat{\alpha}_1 L_t + \hat{\alpha}_2 K_t \qquad t = 1946, \ldots, 1960$

(2) $\hat{Y}_t = \hat{\gamma}_1 L_t + \hat{\gamma}_2 K_t \qquad t = 1961, \ldots, 1980$

Then $\hat{\beta}_3 = \hat{\gamma}_1 - \hat{\alpha}_1$ and $\hat{\beta}_4 = \hat{\gamma}_2 - \hat{\alpha}_2$

For t-tests we need the standard errors of $\hat{\beta}_3$ and $\hat{\beta}_4$,

For $\hat{\beta}_3$:

$$Var(\hat{\beta}_3) = Var(\hat{\gamma}_1) + Var(\hat{\alpha}_1) - 2\underbrace{Cov(\hat{\gamma}_1, \hat{\alpha}_1)}_{=0}$$

(because the variances of disturbance terms are constant in both samples so that the two models are independent from each other)

$$= (0{,}15)^2 + (0{,}21)^2$$
$$= 0{,}0666$$

$$se(\hat{\beta}_3) = \sqrt{Var(\hat{\beta}_3)} = 0{,}258$$

$H_0: \beta_3 = 0$
$H_A: \beta_3 \neq 0$

$$t_{calc} = \frac{0{,}22}{0{,}258} = 0{,}852 < t_{0{,}025,\,35-4} = 2{,}039$$

found in part (a)

DNR $H_0 \Rightarrow$ marginal productivity of labor does not differ between the two subperiods.

For $\hat{\beta}_4$:

$$Var(\hat{\beta}_4) = Var(\hat{\gamma}_2) + Var(\hat{\alpha}_2) - 2\underbrace{Cov(\hat{\gamma}_2, \hat{\alpha}_2)}_{=0}$$

$$= (0{,}30)^2 + (0{,}35)^2$$
$$= 0{,}2125$$

$$Se(\hat{\beta}_4) = \sqrt{Var(\hat{\beta}_4)} = 0{,}4609$$

$$\left.\begin{array}{l} H_0: \beta_4 = 0 \\ H_A: \beta_4 \neq 0 \end{array}\right\} \quad t_{calc} = \frac{0,54}{0,4609} = 1,716 < t_{0,025,31} = 2,039$$

DNR H_0 => marginal productivity does not differ between the two subperiods

Problem 4

a) Here we can only carry out ACOV test between (1) and (3) since model (2) is a single subperiod.
(Individual tests can be carried out using (3) only, and the predictive test will be carried out in (b)).

Our unrestricted model is model (3):

$$\hat{C}_t = \hat{\beta}_0 + \hat{\beta}_1 Y_t + \hat{\beta}_2 L_t + \hat{\beta}_3 \Delta P_t + \hat{\beta}_4 D_t + \hat{\beta}_5 D_t Y_t + \hat{\beta}_6 D_t L_t + \hat{\beta}_7 D_t \Delta P_t$$

H_0: $\beta_4 = \beta_5 = \beta_6 = \beta_7 = 0$ (no structural change)
H_A: at least one differs

Unrestricted model = M3
restricted model = M1

$$Q = \frac{SSR_R - SSR_U}{SSR_U} \times \frac{T - 2(\overset{\text{from the restricted model}}{k+1})}{p} = \frac{0,148 - 0,1316}{0,1316} \times \frac{18-8}{4}$$

$$= 0,311$$

$$0,311 < F_{4,10}^{0,01} = 5,99$$

DNR H_0 => no structural change

c) $\hat{C}_t = \hat{\alpha}_0 + \hat{\alpha}_1 Y_t + \hat{\alpha}_2 L_t + \hat{\alpha}_3 \Delta P_t + \hat{e}_1 I_{78} + \hat{e}_2 I_{79} + \hat{e}_3 I_{80}$

Here we look at the changes in single observations of 78, 79 and 80. The model is used in predictive test where $T < k+1$ (T would be 3 if we took 78-80 as a subperiod).

Therefore, the base parameters will be the same as those obtained from the subsample of 63-77 (model 2).

$\hat{\alpha}_0 = 6{,}29$, $\hat{\alpha}_1 = 0{,}56$, $\hat{\alpha}_2 = 0{,}29$, $\hat{\alpha}_3 = 0{,}47$

Differences in single observations will be captured by the corresponding prediction errors, $\hat{e}_i$ (coefficients of impulse dummies).

Also, SSR of the new model will also be the same as model 2.

So $SSR_U = SSR_{M(2)}$, $SSR_R = SSR_{M(1)}$

Predictive test can be carried out as follows:

$H_0: e_1 = e_2 = e_3 = 0$

H_A: at least one differs

from the unrestricted model

$$Q = \frac{SSR_R - SSR_U}{SSR_U} \times \frac{T-k-1}{p}$$

$$= \frac{0{,}148 - 0{,}137}{0{,}137} \times \frac{11}{3} = 0{,}294 < F^{0.01}_{3,11} = 6{,}21$$

DNR H_0 => no structural change in any of these periods.