

FIRST MIDTERM EXAM FORMULA SHEET

| | |
|---|--|
| $Y_t = \beta_0 + \beta_1 X_t + u_t$ | $\hat{\beta}_1 = \frac{\sum_{t=1}^T x_t y_t}{\sum_{t=1}^T x_t^2}, \quad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$ |
| $Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{t=1}^T x_t^2}$ | $Var(\hat{\beta}_0) = \sigma^2 \frac{\sum_{t=1}^T X_t^2}{T \sum_{t=1}^T x_t^2}$ |
| $Cov(\hat{\beta}_0, \hat{\beta}_1) = \frac{-\bar{X}\sigma^2}{\sum x_t^2}$ | $\hat{\sigma}^2 = \frac{\sum_{t=1}^T \hat{u}_t^2}{T - k - 1}$ |
| $R^2 = 1 - \frac{\sum_{t=1}^T \hat{u}_t^2}{\sum_{t=1}^T y_t^2}$ | $r_{XY} = \frac{\sum_{t=1}^T y_t x_t}{\sqrt{\sum_{t=1}^T x_t^2 \sum_{t=1}^T y_t^2}}$ |
| $r_{Y\hat{Y}} = \frac{\sum_{t=1}^T (\hat{Y}_t - \bar{Y})(Y_t - \bar{Y})}{\sqrt{\sum_{t=1}^T (Y_t - \bar{Y})^2 \sum_{t=1}^T (\hat{Y}_t - \bar{Y})^2}}$ | $R^2 = \hat{\beta}_1^2 \frac{\sum_{t=1}^T x_t^2}{\sum_{t=1}^T y_t^2}, \quad R^2 = \hat{\beta}_1 \frac{\sum_{t=1}^T x_t y_t}{\sum_{t=1}^T y_t^2}$ |
| <i>Correlation between X and Y:</i> $\rho = \frac{Cov(X, Y)}{\sigma_x \sigma_y}$ | $Cov(X, Y) = E\{[X - E(X)][Y - E(Y)]\}$ $Var(Z) = E\{[Z - E(Z)]^2\} = Cov(Z, Z)$ |
| Consider $\mathbf{y} = \mathbf{X}\beta + \mathbf{u}$ $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ $Var(\hat{\beta}) = \sigma^2 (\mathbf{X}'\mathbf{X})^{-1}$ | $Q = \frac{(SSR_R - SSR_U)/p}{SSR_U/(T-k-1)}$ $Q \sim F_{p, T-k-1}^\alpha$ |
| $H_0: \beta_1 = \beta_1^*$ $H_A: \beta_1 \neq \beta_1^*$ $t_{\hat{\beta}_1} = \frac{(\hat{\beta}_1 - \beta_1^*)}{se(\hat{\beta}_1)}$ with $T-k-1$ degrees of freedom | |

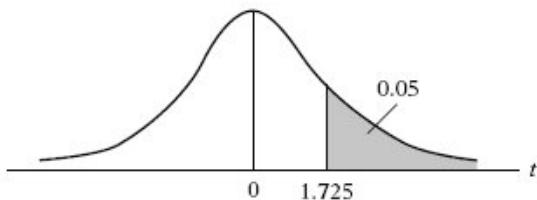
PERCENTAGE POINTS OF THE t DISTRIBUTION

Example

$$\Pr(t > 2.086) = 0.025$$

$$\Pr(t > 1.725) = 0.05 \quad \text{for } df = 20$$

$$\Pr(|t| > 1.725) = 0.10$$

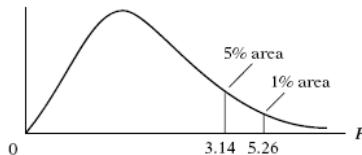


| df | Pr | | | | | | | |
|----------|--------------|--------------|--------------|---------------|--------------|----------------|----------------|--|
| | 0.25 0.50 | 0.10 0.20 | 0.05 0.10 | 0.025 0.05 | 0.01 0.02 | 0.005 0.010 | 0.001 0.002 | |
| 1 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.31 | |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.214 | |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | |

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE D.3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION

Example $\Pr(F > 1.59) = 0.25$ $\Pr(F > 2.42) = 0.10$ for df $N_1 = 10$ $\Pr(F > 3.14) = 0.05$ and $N_2 = 9$ $\Pr(F > 5.26) = 0.01$ 

| df for denominator N_2 | df for numerator N_1 | | | | | | | | | | | | |
|--------------------------|------------------------|------|------|------|------|------|------|------|------|------|------|------|------|
| | Pr | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | .25 | 5.83 | 7.50 | 8.20 | 8.58 | 8.82 | 8.98 | 9.10 | 9.19 | 9.26 | 9.32 | 9.36 | 9.41 |
| | .10 | 39.9 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 58.9 | 59.4 | 59.9 | 60.2 | 60.5 | 60.7 |
| | .05 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 | 243 | 244 |
| 2 | .25 | 2.57 | 3.00 | 3.15 | 3.23 | 3.28 | 3.31 | 3.34 | 3.35 | 3.37 | 3.38 | 3.39 | 3.39 |
| | .10 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 | 9.40 | 9.41 |
| | .05 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 |
| 3 | .01 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 |
| | .25 | 2.02 | 2.28 | 2.36 | 2.39 | 2.41 | 2.42 | 2.43 | 2.44 | 2.44 | 2.44 | 2.45 | 2.45 |
| | .10 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.23 | 5.22 | 5.22 |
| 4 | .05 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.76 | 8.74 |
| | .01 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.2 | 27.1 | 27.1 |
| | .25 | 1.81 | 2.00 | 2.05 | 2.06 | 2.07 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 |
| 5 | .10 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 | 3.92 | 3.91 | 3.90 |
| | .05 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.94 | 5.91 |
| | .01 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.5 | 14.4 | 14.4 |
| 6 | .25 | 1.69 | 1.85 | 1.88 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 |
| | .10 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 | 3.28 | 3.27 |
| | .05 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.71 | 4.68 |
| 7 | .01 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 | 9.96 | 9.89 |
| | .25 | 1.62 | 1.76 | 1.78 | 1.79 | 1.79 | 1.78 | 1.78 | 1.77 | 1.77 | 1.77 | 1.77 | 1.77 |
| | .10 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.94 | 2.92 | 2.90 |
| 8 | .05 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.03 | 4.00 |
| | .01 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.79 | 7.72 |
| | .25 | 1.57 | 1.70 | 1.72 | 1.71 | 1.71 | 1.70 | 1.69 | 1.69 | 1.69 | 1.68 | 1.68 | 1.68 |
| 9 | .10 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.70 | 2.68 | 2.67 |
| | .05 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.60 | 3.57 |
| | .01 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.54 | 6.47 |
| 10 | .25 | 1.54 | 1.66 | 1.67 | 1.66 | 1.66 | 1.65 | 1.64 | 1.64 | 1.63 | 1.63 | 1.62 | 1.62 |
| | .10 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 | 2.54 | 2.52 | 2.50 |
| | .05 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.31 | 3.28 |
| 11 | .01 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.73 | 5.67 |
| | .25 | 1.51 | 1.62 | 1.63 | 1.63 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.58 |
| | .10 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 | 2.42 | 2.40 | 2.38 |
| 12 | .05 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.10 | 3.07 |
| | .01 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.18 | 5.11 |

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

| df for denominator N_2 | df for numerator N_1 | | | | | | | | | | | | df for denominator N_2 |
|--------------------------|------------------------|------|------|------|------|------|------|------|------|------|------|----------|--------------------------|
| | 15 | 20 | 24 | 30 | 40 | 50 | 60 | 100 | 120 | 200 | 500 | ∞ | |
| 1 | 9.49 | 9.58 | 9.63 | 9.67 | 9.71 | 9.74 | 9.76 | 9.78 | 9.80 | 9.82 | 9.84 | 9.85 | .25 |
| | 61.2 | 61.7 | 62.0 | 62.3 | 62.5 | 62.7 | 62.8 | 63.0 | 63.1 | 63.2 | 63.3 | 63.3 | .10 |
| | 246 | 248 | 249 | 250 | 251 | 252 | 252 | 253 | 253 | 254 | 254 | 254 | .05 |
| 2 | 3.41 | 3.43 | 3.43 | 3.44 | 3.45 | 3.45 | 3.46 | 3.47 | 3.47 | 3.48 | 3.48 | 3.48 | .25 |
| | 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.47 | 9.47 | 9.48 | 9.48 | 9.49 | 9.49 | 9.49 | .10 |
| | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | .05 |
| 3 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | .01 |
| | 2.46 | 2.46 | 2.46 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | .25 |
| | 5.20 | 5.18 | 5.18 | 5.17 | 5.16 | 5.15 | 5.15 | 5.14 | 5.14 | 5.14 | 5.14 | 5.13 | .10 |
| 4 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.58 | 8.57 | 8.55 | 8.55 | 8.54 | 8.53 | 8.53 | .05 |
| | 26.9 | 26.7 | 26.6 | 26.5 | 26.4 | 26.4 | 26.3 | 26.2 | 26.2 | 26.1 | 26.1 | 26.1 | .01 |
| | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | .25 |
| 5 | 3.87 | 3.84 | 3.83 | 3.82 | 3.80 | 3.80 | 3.79 | 3.78 | 3.78 | 3.77 | 3.76 | 3.76 | .10 |
| | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.70 | 5.69 | 5.66 | 5.66 | 5.65 | 5.64 | 5.63 | .05 |
| | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 | 13.7 | 13.6 | 13.6 | 13.6 | 13.5 | 13.5 | 13.5 | .01 |
| 6 | 1.89 | 1.88 | 1.88 | 1.88 | 1.87 | 1.87 | 1.87 | 1.87 | 1.87 | 1.87 | 1.87 | 1.87 | .25 |
| | 3.24 | 3.21 | 3.19 | 3.17 | 3.16 | 3.15 | 3.14 | 3.13 | 3.12 | 3.12 | 3.11 | 3.10 | .10 |
| | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.44 | 4.43 | 4.41 | 4.40 | 4.39 | 4.37 | 4.36 | .05 |
| 7 | 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.24 | 9.20 | 9.13 | 9.11 | 9.08 | 9.04 | 9.02 | .01 |
| | 1.76 | 1.76 | 1.75 | 1.75 | 1.75 | 1.75 | 1.74 | 1.74 | 1.74 | 1.74 | 1.74 | 1.74 | .25 |
| | 2.87 | 2.84 | 2.82 | 2.80 | 2.78 | 2.77 | 2.76 | 2.75 | 2.74 | 2.73 | 2.73 | 2.72 | .10 |
| 8 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.75 | 3.74 | 3.71 | 3.70 | 3.69 | 3.68 | 3.67 | .05 |
| | 5.76 | 7.40 | 7.31 | 7.23 | 7.14 | 7.09 | 7.06 | 6.99 | 6.97 | 6.93 | 6.90 | 6.88 | .01 |
| | 1.68 | 1.67 | 1.66 | 1.66 | 1.66 | 1.66 | 1.65 | 1.65 | 1.65 | 1.65 | 1.65 | 1.65 | .25 |
| 9 | 2.63 | 2.59 | 2.58 | 2.56 | 2.54 | 2.52 | 2.51 | 2.50 | 2.49 | 2.48 | 2.48 | 2.47 | .10 |
| | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.32 | 3.30 | 3.27 | 3.27 | 3.25 | 3.24 | 3.23 | .05 |
| | 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.86 | 5.82 | 5.75 | 5.74 | 5.70 | 5.67 | 5.65 | .01 |
| 10 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.58 | 1.58 | 1.58 | 1.58 | 1.58 | .25 |
| | 2.46 | 2.42 | 2.40 | 2.38 | 2.36 | 2.35 | 2.34 | 2.32 | 2.32 | 2.31 | 2.30 | 2.29 | .10 |
| | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.02 | 3.01 | 2.97 | 2.97 | 2.95 | 2.94 | 2.93 | .05 |
| 11 | 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.07 | 5.03 | 4.96 | 4.95 | 4.91 | 4.88 | 4.86 | .01 |
| | 1.57 | 1.56 | 1.56 | 1.55 | 1.55 | 1.54 | 1.54 | 1.53 | 1.53 | 1.53 | 1.53 | 1.53 | .25 |
| | 2.34 | 2.30 | 2.28 | 2.25 | 2.23 | 2.22 | 2.21 | 2.19 | 2.18 | 2.17 | 2.17 | 2.16 | .10 |
| 12 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.80 | 2.79 | 2.75 | 2.75 | 2.73 | 2.72 | 2.71 | .05 |
| | 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.52 | 4.48 | 4.42 | 4.40 | 4.36 | 4.33 | 4.31 | .01 |

(Continued)

TABLE D.3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION (Continued)

| df for denominator N ₂ | df for numerator N ₁ | | | | | | | | | | | | df for denominator N ₂ | |
|-----------------------------------|---------------------------------|------|------|------|------|------|------|------|------|------|------|------|-----------------------------------|----|
| | Pr | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 10 | .25 | 1.49 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.57 | 1.56 | 1.56 | 1.55 | 1.55 | 1.54 | 10 |
| | .10 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 | 2.30 | 2.28 | |
| | .05 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 | |
| | .01 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.77 | 4.71 | |
| 11 | .25 | 1.47 | 1.58 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 | 1.53 | 1.52 | 1.52 | 1.51 | 11 |
| | .10 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 | 2.23 | 2.21 | |
| | .05 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.82 | 2.79 | |
| | .01 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.46 | 4.40 | |
| 12 | .25 | 1.46 | 1.56 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 | 1.50 | 1.50 | 1.49 | 1.49 | 12 |
| | .10 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 | 2.19 | 2.17 | 2.15 | |
| | .05 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.72 | 2.69 | |
| | .01 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.22 | 4.16 | |
| 13 | .25 | 1.45 | 1.55 | 1.55 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.47 | 13 |
| | .10 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 | 2.14 | 2.12 | 2.10 | |
| | .05 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.63 | 2.60 | |
| | .01 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 4.02 | 3.96 | |
| 14 | .25 | 1.44 | 1.53 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.46 | 1.45 | 14 |
| | .10 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 | 2.10 | 2.08 | 2.05 | |
| | .05 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.57 | 2.53 | |
| | .01 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.86 | 3.80 | |
| 15 | .25 | 1.43 | 1.52 | 1.52 | 1.51 | 1.49 | 1.48 | 1.47 | 1.46 | 1.46 | 1.45 | 1.44 | 1.44 | 15 |
| | .10 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 | 2.04 | 2.02 | |
| | .05 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.51 | 2.48 | |
| | .01 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.73 | 3.67 | |
| 16 | .25 | 1.42 | 1.51 | 1.51 | 1.50 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.44 | 1.43 | 1.43 | 16 |
| | .10 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 | 2.03 | 2.01 | 1.99 | |
| | .05 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.46 | 2.42 | |
| | .01 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.62 | 3.55 | |
| 17 | .25 | 1.42 | 1.51 | 1.50 | 1.49 | 1.47 | 1.46 | 1.45 | 1.44 | 1.44 | 1.43 | 1.43 | 1.43 | 17 |
| | .10 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 | 2.00 | 1.98 | 1.96 | |
| | .05 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.41 | 2.38 | |
| | .01 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.52 | 3.46 | |
| 18 | .25 | 1.41 | 1.50 | 1.49 | 1.48 | 1.46 | 1.45 | 1.44 | 1.43 | 1.43 | 1.42 | 1.41 | 1.40 | 18 |
| | .10 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.98 | 1.96 | 1.93 | |
| | .05 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.37 | 2.34 | |
| | .01 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.43 | 3.37 | |
| 19 | .25 | 1.41 | 1.49 | 1.49 | 1.47 | 1.46 | 1.44 | 1.43 | 1.42 | 1.41 | 1.41 | 1.40 | 1.40 | 19 |
| | .10 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 | 1.96 | 1.94 | 1.91 | |
| | .05 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.34 | 2.31 | |
| | .01 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.36 | 3.30 | |
| 20 | .25 | 1.40 | 1.49 | 1.48 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.39 | 20 |
| | .10 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 | 1.92 | 1.89 | |
| | .05 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.22 | |
| | .01 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.29 | 3.23 | |

(Continued)

TABLE D.3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION (Continued)

| df for denominator N_2 | df for numerator N_1 | | | | | | | | | | | | df for numerator N_1 | | | | | | | | | | | df for denominator N_2 | | |
|--------------------------|------------------------|------|------|------|------|------|------|------|------|------|------|------|------------------------|------|------|------|------|------|------|------|------|------|------|--------------------------|----------|-----|
| | Pr | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 15 | 20 | 24 | 30 | 40 | 50 | 60 | 100 | 120 | 200 | 500 | ∞ | Pr |
| 22 | .25 | 1.40 | 1.48 | 1.47 | 1.45 | 1.44 | 1.42 | 1.41 | 1.40 | 1.39 | 1.39 | 1.38 | 1.37 | 1.36 | 1.34 | 1.33 | 1.32 | 1.31 | 1.31 | 1.30 | 1.30 | 1.30 | 1.29 | 1.29 | 1.28 | .25 |
| | .10 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 | 1.90 | 1.88 | 1.86 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 | .10 |
| | .05 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.26 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.91 | 1.89 | 1.85 | 1.84 | 1.82 | 1.80 | 1.78 | .05 |
| | .01 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.18 | 3.12 | 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.53 | 2.50 | 2.42 | 2.40 | 2.36 | 2.33 | 2.31 | .01 |
| 24 | .25 | 1.39 | 1.47 | 1.46 | 1.44 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.38 | 1.37 | 1.36 | 1.35 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.28 | 1.27 | 1.27 | 1.26 | .25 |
| | .10 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 | 1.88 | 1.85 | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.62 | 1.61 | 1.58 | 1.57 | 1.56 | 1.54 | 1.53 | .10 |
| | .05 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.21 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.86 | 1.84 | 1.80 | 1.79 | 1.77 | 1.75 | 1.73 | .05 |
| | .01 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.09 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.44 | 2.40 | 2.33 | 2.31 | 2.27 | 2.24 | 2.21 | .01 |
| 26 | .25 | 1.38 | 1.46 | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.37 | 1.36 | 1.35 | 1.34 | 1.34 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.28 | 1.27 | 1.27 | 1.26 | 1.25 | .25 |
| | .10 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 | 1.86 | 1.84 | 1.81 | 1.76 | 1.71 | 1.68 | 1.65 | 1.61 | 1.59 | 1.58 | 1.55 | 1.54 | 1.53 | 1.50 | 1.49 | .10 |
| | .05 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.18 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.82 | 1.80 | 1.76 | 1.75 | 1.73 | 1.71 | 1.69 | .05 |
| | .01 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 3.02 | 2.96 | 2.81 | 2.66 | 2.58 | 2.50 | 2.42 | 2.36 | 2.33 | 2.25 | 2.23 | 2.19 | 2.16 | 2.13 | .01 |
| 28 | .25 | 1.38 | 1.46 | 1.45 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.28 | 1.26 | 1.26 | 1.26 | 1.25 | 1.25 | 1.25 | .25 |
| | .10 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 | 1.84 | 1.81 | 1.79 | 1.74 | 1.69 | 1.66 | 1.63 | 1.59 | 1.57 | 1.56 | 1.53 | 1.52 | 1.50 | 1.49 | 1.48 | .10 |
| | .05 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.15 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.79 | 1.77 | 1.73 | 1.71 | 1.69 | 1.67 | 1.65 | .05 |
| | .01 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.96 | 2.90 | 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.30 | 2.26 | 2.19 | 2.17 | 2.13 | 2.09 | 2.06 | .01 |
| 30 | .25 | 1.38 | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.34 | 1.32 | 1.30 | 1.29 | 1.28 | 1.27 | 1.27 | 1.26 | 1.25 | 1.25 | 1.24 | 1.24 | 1.23 | .25 |
| | .10 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 | 1.79 | 1.77 | 1.72 | 1.67 | 1.64 | 1.61 | 1.57 | 1.55 | 1.54 | 1.51 | 1.50 | 1.48 | 1.47 | 1.46 | .10 |
| | .05 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.76 | 1.74 | 1.70 | 1.68 | 1.66 | 1.64 | 1.62 | .05 |
| | .01 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.91 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.25 | 2.21 | 2.13 | 2.11 | 2.07 | 2.03 | 2.01 | .01 |
| 40 | .25 | 1.36 | 1.44 | 1.42 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.28 | 1.27 | 1.26 | 1.25 | 1.24 | 1.24 | 1.23 | 1.23 | 1.22 | 1.21 | 1.20 | .25 |
| | .10 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 | 1.73 | 1.71 | 1.66 | 1.61 | 1.57 | 1.54 | 1.51 | 1.48 | 1.47 | 1.43 | 1.42 | 1.41 | 1.39 | 1.38 | .10 |
| | .05 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.66 | 1.64 | 1.59 | 1.58 | 1.55 | 1.53 | 1.51 | .05 |
| | .01 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.73 | 2.66 | 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.06 | 2.02 | 1.94 | 1.92 | 1.87 | 1.83 | 1.80 | .01 |
| 60 | .25 | 1.35 | 1.42 | 1.41 | 1.38 | 1.37 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.29 | 1.27 | 1.25 | 1.24 | 1.22 | 1.21 | 1.20 | 1.19 | 1.17 | 1.17 | 1.16 | 1.15 | 1.15 | .25 |
| | .10 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 | 1.68 | 1.66 | 1.60 | 1.54 | 1.51 | 1.48 | 1.44 | 1.41 | 1.40 | 1.36 | 1.35 | 1.33 | 1.31 | 1.29 | .10 |
| | .05 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.56 | 1.53 | 1.48 | 1.47 | 1.44 | 1.41 | 1.39 | .05 |
| | .01 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.56 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.88 | 1.84 | 1.75 | 1.73 | 1.68 | 1.63 | 1.60 | .01 |
| 120 | .25 | 1.34 | 1.40 | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 | 1.24 | 1.22 | 1.21 | 1.19 | 1.18 | 1.17 | 1.16 | 1.15 | 1.15 | 1.15 | 1.15 | 1.15 | .25 |
| | .10 | 2.75 | 2.35 | 2.13 | 1.99 | 1.90 | 1.82 | 1.77 | 1.72 | 1.68 | 1.65 | 1.62 | 1.60 | 1.55 | 1.48 | 1.45 | 1.41 | 1.37 | 1.34 | 1.32 | 1.27 | 1.26 | 1.24 | 1.21 | 1.19 | .10 |
| | .05 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.09 | 2.02 | 1.96 | 1.91 | 1.87 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.46 | 1.43 | 1.37 | 1.35 | 1.32 | 1.28 | 1.25 | .05 |
| | .01 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.40 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.70 | 1.66 | 1.56 | 1.53 | 1.48 | 1.42 | 1.38 | .01 |
| 200 | .25 | 1.33 | 1.39 | 1.38 | 1.36 | 1.34 | 1.32 | 1.31 | 1.29 | 1.28 | 1.27 | 1.26 | 1.25 | 1.23 | 1.21 | 1.20 | 1.18 | 1.16 | 1.14 | 1.12 | 1.11 | 1.10 | 1.09 | 1.08 | 1.06 | .25 |
| | .10 | 2.73 | 2.33 | 2.11 | 1.97 | 1.88 | 1.80 | 1.75 | 1.70 | 1.66 | 1.63 | 1.60 | 1.57 | 1.52 | 1.46 | 1.42 | 1.38 | 1.34 | 1.31 | 1.28 | 1.24 | 1.22 | 1.20 | 1.17 | 1.14 | .10 |
| | .05 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.84 | 1.80 | 1.72 | 1.62 | 1.57 | 1.52 | 1.46 | 1.41 | 1.39 | 1.32 | 1.29 | 1.26 | 1.22 | 1.19 | .05 |
| | .01 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.34 | 2.27 | 2.13 | 1.97 | 1.89 | 1.79 | 1.69 | 1.63 | 1.58 | 1.48 | 1.44 | 1.39 | 1.33 | 1.28 | .01 |
| ∞ | .25 | 1.32 | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.29 | 1.28 | 1.27 | 1.25 | 1.24 | 1.24 | 1.22 | 1.19 | 1.18 | 1.16 | 1.14 | 1.13 | 1.12 | 1.09 | 1.08 | 1.07 | 1.04 | 1.00 | .25 |
| | .10 | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 | 1.60 | 1.57 | 1.55 | 1.49 | 1.42 | 1.38 | 1.34 | 1.30 | 1.26 | 1.24 | 1.18 | 1.17 | 1.13 | 1.08 | 1.00 | .10 |
| | .05 | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.79 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.35 | 1.32 | 1.24 | 1.22 | 1.17 | 1.11 | 1.00 | .05 |
| | .01 | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.25 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.52 | 1.47 | 1.36 | 1.32 | 1.25 | 1.15 | 1.00 | .01 |