

LECTURE NOTE 03

ELASTICITY

Outline of today's course:

(1) Elasticity

I. Elasticity

- Suppose $P_x=1$, $Q_x=10$, Total revenue is $TR=1 \cdot 10=10$.
- Think two different societies.

1. Suppose further that if price of X increased to $P_x=1.1$ then $Q_x=8.5$

- In this case $TR=1.1 \times 8.5=9.35$
- Hence, in this case TR decreased. WHY?
 - Since, 10 % price increase resulted to a 15 % decrease in quantity demanded (a high response).

$$\begin{array}{ccc} P_x & \cdot & Q_x & = & TR \\ (10 \% \uparrow) & & (15 \% \downarrow) & & (6.5 \% \downarrow) \end{array}$$

2. Now suppose that if that if price of X increased to $P_x=1.1$ then $Q_x=9.5$

- In this case $TR=1.1 \times 9.5=10.45$
- Hence, in this case TR increased. WHY?
 - Since, 10 % price increase resulted to *only* a 5 % decrease in quantity demanded (a small response).

$$\begin{array}{ccc} P_x & \cdot & Q_x & = & TR \\ (10 \% \uparrow) & & (5 \% \downarrow) & & (0.45 \% \uparrow) \end{array}$$

Conclusion: *The ratio of percentage changes (in prices and quantities) is determinative for the impact on Total Revenue (TR).*

- We need a general measure.
- This is known as “Elasticity” in economics.
- Elasticity can be used to measure the responsiveness of anything to anything else.
- **Elasticity** is a general concept that can be used to quantify the response in one variable when another variable changes.

$$\text{Elasticity of A with respect to B} = \frac{\% \Delta A}{\% \Delta B}$$

Price Elasticity of Demand

Price Elasticity of Demand = (% change in the quantity demanded / % change in the price of x)

$$\varepsilon = -\frac{\frac{\Delta Q_x}{Q_x}}{\frac{\Delta P_x}{P_x}}$$

- **Thus, elasticity of demand measures price responsiveness of demand.**
- **Because** price and quantity demanded are inversely related, the price elasticity of demand has a negative sign.
- Elasticity is *independent of units*.
- For our examples

$$1. \frac{\Delta Q_x}{Q_x} = (8.5 - 10) / 10 = -0.15$$

$$\frac{\Delta P_x}{P_x} = (1.1 - 1) / 1 = 0.1$$

$$\text{Hence } \varepsilon = -\frac{\frac{\Delta Q_x}{Q_x}}{\frac{\Delta P_x}{P_x}} = \frac{-0.15}{0.1} = -1.5$$

1 % increase in price will result in a 1.5 % fall in quantity demanded

$$2. \frac{\Delta Q_x}{Q_x} = (9.5 - 10) / 10 = -0.05$$

$$\frac{\Delta P_x}{P_x} = (1.1 - 1) / 1 = 0.1$$

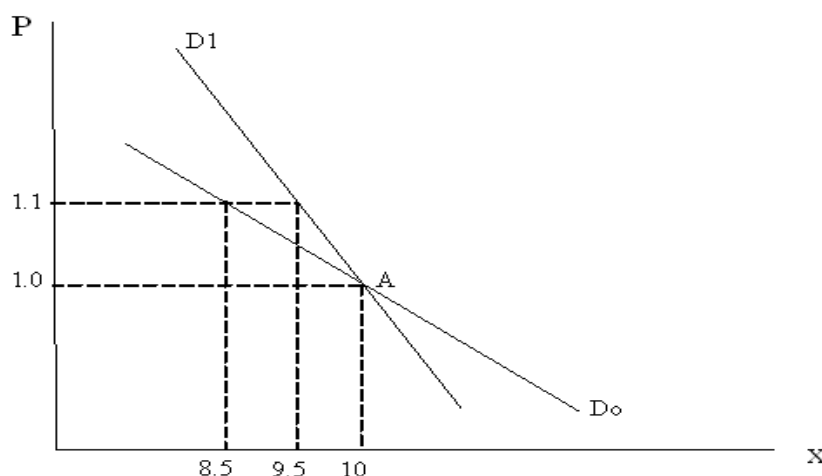
$$\text{Hence } \varepsilon = \frac{\frac{\Delta Q_x}{Q_x}}{\frac{\Delta P_x}{P_x}} = \frac{-0.05}{0.1} = -0.5$$

1 % increase in price will result in a 0.5 % fall in quantity demanded.

- Thus, in the first case, the price elasticity is higher in absolute values.

○ This means that in the first case, the quantity demanded changes more in response to a change in prices, in percentage terms.

- Let us draw the corresponding demand curves for our two examples:



- We know that the elasticity of demand along D0 as measured at

point A is -1.5 whereas the elasticity of demand along D1 as measured at point A is -0.5.

- Hence, the demand curve D1 is more elastic than D0 at point A.
- Why? →

Note that the formal definition of (*point*) elasticity of demand is as follows:

$$\varepsilon = -\frac{\frac{\partial Q_x}{Q_x}}{\frac{\partial P_x}{P_x}}$$

which yields

$$\varepsilon = -\frac{\frac{\partial Q_x}{Q_x}}{\frac{\partial P_x}{P_x}} = \frac{\partial Q_x}{\partial P_x} \cdot \frac{P_x}{Q_x}$$

where $\frac{\partial Q_x}{\partial P_x}$ is the *reciprocal* of the slope of the demand curve.

For two intersecting demand curves;

- *At any intersection point (P*,Q*), the flatter demand curve (with lower slope) has higher elasticity at that point.*
- *Flatness and elasticity are closely related concepts.*

Elasticity and Revenue

- As our examples show
 - the net effect of a change in price on revenue depends on the price elasticity of demand.

- Let us analyze the relationship among elasticity and TR.

○ $TR = P \cdot Q$ and $Q = f(P)$

$$\frac{dTR}{dP} = Q + P \left(\frac{dQ}{dP} \right)$$

$$\frac{dTR}{dP} = Q \left(1 + \underbrace{\frac{P}{Q} \cdot \frac{dQ}{dP}}_{\varepsilon} \right)$$

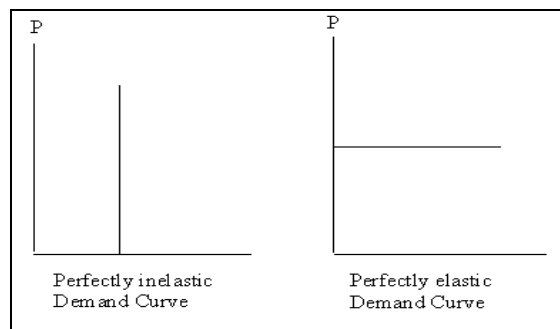
$$\frac{dTR}{dP} = Q(1 - |\varepsilon|)$$

Hence, as a result of a price increase:

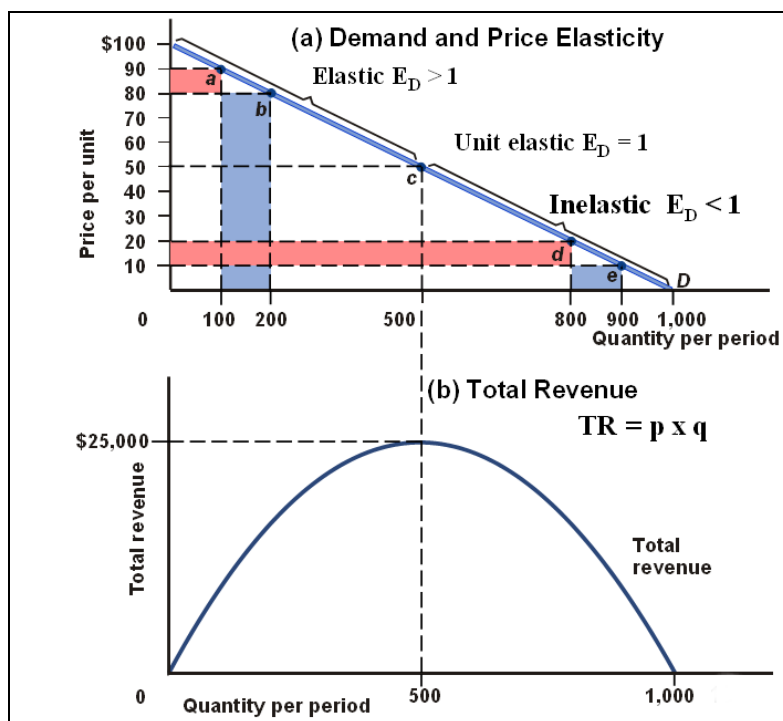
- Total revenue falls ($dTR/dP < 0$) when the elasticity of demand (ε) is higher than 1 in absolute values,
- TR remains constant ($dTR/dP = 0$) when ε is equal to 1 in absolute values,
- TR increases ($dTR/dP > 0$) when ε is lower than 1 in absolute values.
- TR increases and $dTR/dP = Q$ when ε is zero.
 - This means that Q is not a function of price in this case. Consumer purchases do not respond at all to any change in price.
- TR decreases infinitely ($dTR/dP \rightarrow -\infty$) when ε is ∞
 - Consumers will purchase as much as they want at the going price, but only at that price.

Now we can draw the following conclusions;

Numerical Value of ε	Term Used
$\varepsilon = 0$	Perfectly (or completely) inelastic
$\varepsilon < 1$	Inelastic
$\varepsilon = 1$	Unit elastic
$\varepsilon > 1$	Elastic
$\varepsilon = \infty$	Perfectly (or infinitely) elastic



- **Linear demand curves do not have constant elasticities** along the curve.
- Although the slope of a straight line demand curve remains constant throughout its length, its elasticity does not.
- Along a straight-line demand curve, the price elasticity of demand decreases as you move from left to right.



Not: $a, d \rightarrow$ red boxes (losses in TR); $b, e \rightarrow$ blue boxes (gains in TR)

When demand is elastic,

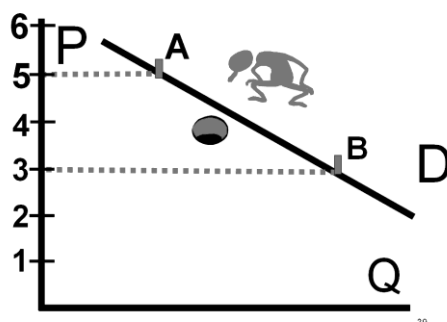
- a decrease in price (from a to b) will increase total revenue
 - because the gain in revenue from selling more units (blue box) exceeds the loss in revenue from selling all units at a lower price (red box)

When demand is inelastic,

- a price decrease (from d to e) reduces total revenue
 - because the gain in revenue from selling more units (blue box) is less than the loss in revenue at the lower price (red box)

Arc Elasticity of Demand (Midpoint Formula)

Problem: When you move along a demand curve between two points, you will get different answers to elasticity depending if you are moving up or down the demand curve



- If you go from 3 to 5, the percentage change is $2/3$, but if you go from 5 to 3, the percentage change is $2/5$, so the elasticities are different !
- The answer to this problem is to use the arc elasticity of demand formula which is ...

$$\text{Arc Elas} = \frac{\frac{\text{change in the quantity demanded}}{\text{sum of quantities}/2}}{\frac{\text{change in price}}{\text{sum of prices}/2}}$$

- Or, more formally as

$$\epsilon_{arc} = \frac{\frac{q' - q}{(q' + q)/2}}{\frac{p' - p}{(p' + p)/2}}$$

Determinants of Demand Elasticity

- *Availability of substitutes*
 - demand is more elastic when there are more substitutes for the product.
- *Importance of the item in the budget*
 - demand is more elastic when the item is a more significant portion of the consumer's budget.
- *Time dimension*
 - demand becomes more elastic over time.
 - Since households make adjustments over time and producers develop substitute goods.

Other Important Elasticities

(1) Cross-Price Elasticity of Demand

- Percentage change in the demand of one good divided by the percentage change in the price of another good.

$$\text{Cross Price Elasticity of Demand} = \frac{\% \text{ change in quantity demanded of } X}{\% \text{ change in price of } Y}$$

- If an increase in the price of one good leads to an increase in the demand for another good, their cross-price elasticity is positive → the two goods are *substitutes*.
- If an increase in the price of one good leads to a decrease in

the demand for another, their cross-price elasticity is negative
→ the two goods are *complements*

- Higher the value of (positive) cross price elasticity, the higher the substitubility between the goods.
- X and Y are *perfect substitutes* as the cross-price elasticity between them approaches infinity.
 - In this case they must have the same price.

(2) Income Elasticity of Demand

- It measures the responsiveness of demand to changes in income.

$$\text{Income elasticity of demand} = \frac{\% \text{ change in quantity demanded}}{\% \text{ change in income}}$$

(3) Elasticity of supply

- It is a measure of the response of *quantity of a good supplied* to a *change in price of that good*.
- Likely to be *positive* in output markets

$$\text{Elasticity of supply} = \frac{\% \text{ change in quantity supplied}}{\% \text{ change in price}}$$

For example, take the case of *elasticity of labor supply*.

Elasticity of labor supply: A measure of the response of labor supplied to a change in the price of labor.

$$\text{Elasticity of labor supply} = \frac{\% \text{ change in quantity of labor supplied}}{\% \text{ change in the wage rate}}$$