HANDOUT 05 (REVISED)

TIME SERIES ANALYSIS III

Outline of this lecture:

I. Some Words of Warning about DF Tests

Simulation studies have shown that the DF and ADF tests are lack of power. Power measures the ability of a test to detect a false null hypothesis. In DF and ADF tests, this means the power to detect stationarity.

In other words, lack of such power implies that Y_t may be stationary but the DF tests may fail to detect this. This has been found to be particularly the case when the time series, although stationary, are close to being nonstationary. That is if δ is negative but very close to zero, then DF tests will often fail to reject $H_0 : \delta = 0$ in favor of $H_0 : \delta < 0$.

In addition, the critical values of DF tables have been obtained by simulations and therefore they are only approximate. Also, it is sometime unclear how many lags should be added in ADF regressions. Since the number of lags can seriously affect the value of ADF statistics, this is a further source of uncertainty.

Dickey-Fuller tests should therefore be applied with care, their results being interpreted cautiously. Other sources of information, such as the *correlogram* for the time series, *should not be ignored*.

Even then, it may sometimes be the case that the order of integration of a time series is unclear. For example, it is often uncertain whether price series are I(1) or I(2).

II. Testing Joint Hypothesis with the Dickey-Fuller Tests

Dickey and Fuller (1981) provided tests for testing jointly the parameters a_0, a_2 and δ.

Consider again:

(1) Pure random walk

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(2) Random Walk with Drift

$$\Delta Y_t = a_0 + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(3) Random Walk with Drift and Deterministic Trend

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

<u>Using (2)</u>

- $H_0 : a_0 = \delta = 0$ can be tested using $F(\phi_1)$ statistic.

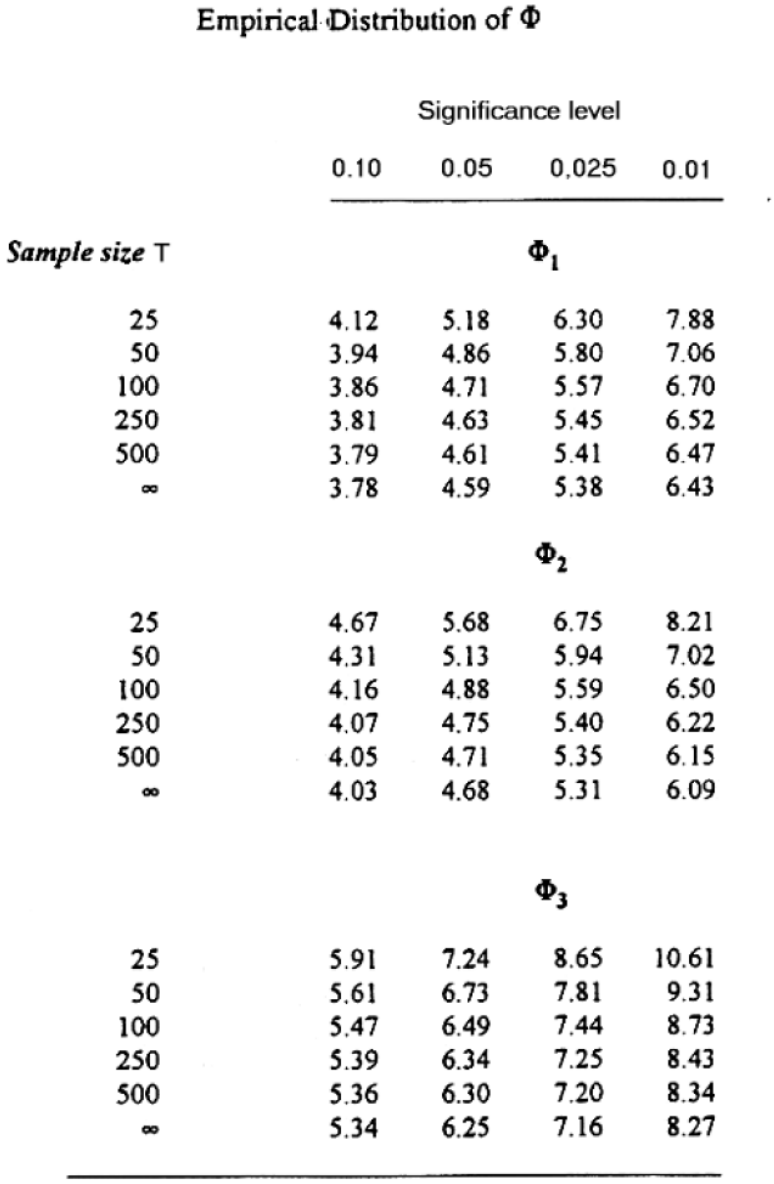
<u>Using (3)</u>

- $H_0 : a_0 = a_2 = \delta = 0$ can be tested using $F(\phi_2)$ statistic.
- $H_0 : a_2 = \delta = 0$ can be tested using $F(\phi_3)$ statistic.

Here; $F(\phi_1)$, $F(\phi_2)$ and $F(\phi_3)$ statistics are constructed in exactly the same way as usual F tests: $F(\phi_i) = \dfrac{(SSR_R - SSR_U)/p}{SSR_U\ /T-k-1}$

where P is the number of restrictions and T-k-1 is the degrees of freedom in unrestricted model. The critical values are simulated by Dickey and Fuller as ϕ_1, ϕ_2 and ϕ_3.

If $F(\phi_i) > \phi_i$, then we reject the null hypothesis as usual.



Empirical Distribution of Φ

	Significance level			
	0.10	0.05	0.025	0.01
Sample size T		Φ_1		
25	4.12	5.18	6.30	7.88
50	3.94	4.86	5.80	7.06
100	3.86	4.71	5.57	6.70
250	3.81	4.63	5.45	6.52
500	3.79	4.61	5.41	6.47
∞	3.78	4.59	5.38	6.43
		Φ_2		
25	4.67	5.68	6.75	8.21
50	4.31	5.13	5.94	7.02
100	4.16	4.88	5.59	6.50
250	4.07	4.75	5.40	6.22
500	4.05	4.71	5.35	6.15
∞	4.03	4.68	5.31	6.09
		Φ_3		
25	5.91	7.24	8.65	10.61
50	5.61	6.73	7.81	9.31
100	5.47	6.49	7.44	8.73
250	5.39	6.34	7.25	8.43
500	5.36	6.30	7.20	8.34
∞	5.34	6.25	7.16	8.27

III. DF and ADF Testing Procedures

Up to now we have seen 3 ADF tests:

(1) Pure random walk

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(2) Random Walk with Drift

$$\Delta Y_t = a_0 + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(3) Random Walk with Drift and Deterministic Trend

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

Also we have seen 3 DF tests:

(1) Pure random walk

$$\Delta Y_t = \delta Y_{t-1} + u_t$$

(2) Random Walk with Drift

$$\Delta Y_t = a_0 + \delta Y_{t-1} + u_t$$

(3) Random Walk with Drift and Deterministic Trend

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + u_t$$

First of all we will start with a DF test. We will check if there is autocorrelation (AC) in DF regression. If there is AC in DF regression then we move to ADF testing.

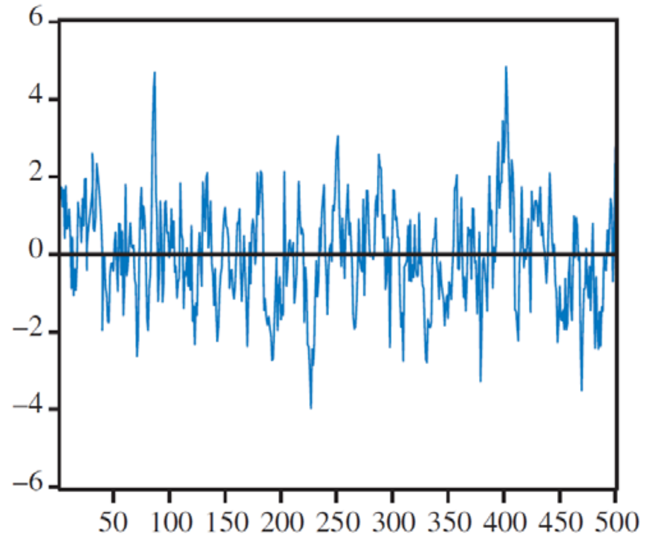
However, for all cases, there is a question concerning whether it is most appropriate to estimate the DF and/or ADF regression in

(1) Pure random walk (version 1)
(2) Random walk with drift (version 2)
(3) Random walk with drift and deterministic trend (version 3)

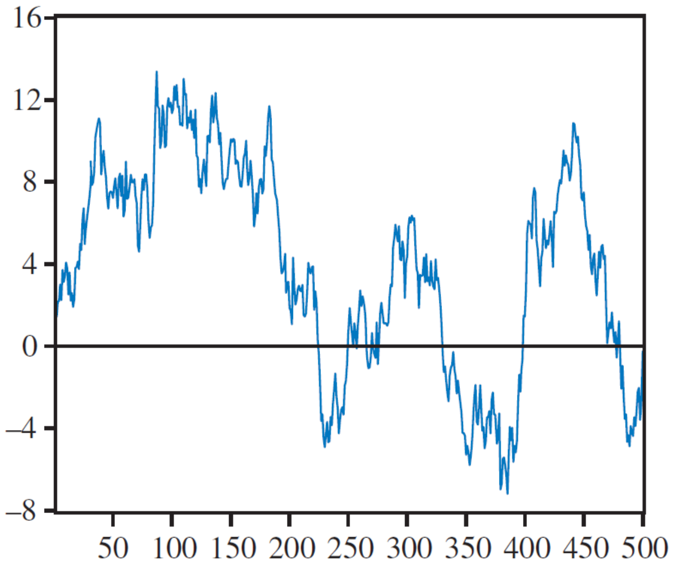
How do we go about deciding which version to use?

First Step: Visual Inspection Plot the time series Y_t and select a suitable DF test based on a visual inspection of the plot.

(1) If the series appears to be wandering or fluctuating around a sample average of zero, use test equation "*<u>version 1</u>*".

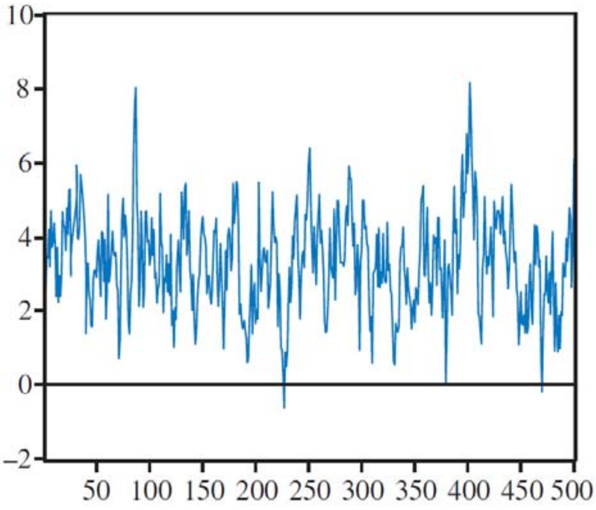


(*a*) $y_t = 0.7y_{t-1} + v_t$

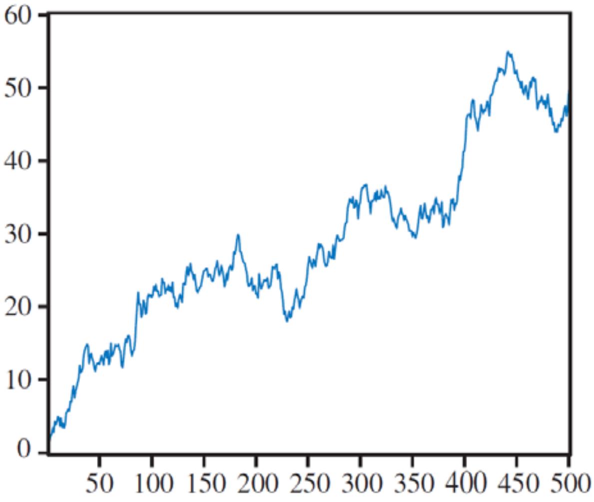


(*d*) $y_t = y_{t-1} + v_t$

(2) If the series appears to be wandering or fluctuating *around a sample average which is nonzero*, use test equation "*version 2*"



$(b)\ y_t = 1 + 0.7y_{t-1} + v_t$

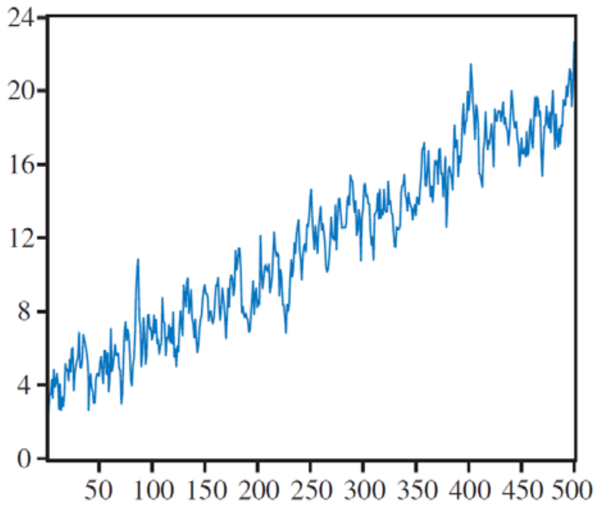


$(e)\ y_t = 0.1 + y_{t-1} + v_t$

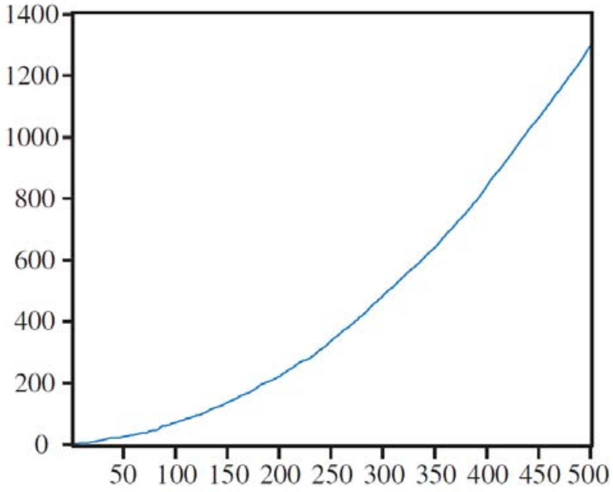
This formulation is appropriate for non-trending financial series like interest rate and exchange rates.

(3) If the series appears to be wandering or fluctuating *around a linear trend* use test equation "*version 3*".

This formulation is appropriate for trending time series like asset prices, or the levels of macroeconomic aggregates like real GDP.



(c) $y_t = 1 + 0.01t + 0.7y_{t-1} + v_t$



(f) $y_t = 0.1 + 0.01t + y_{t-1} + v_t$

Note that plotting the data and observing the graph is sometimes very useful since it can indicate clearly the presence or not of *deterministic* regressors: *drift* and *deterministic trend*.

Second Step: Theory Theoretical considerations might suggest the appropriate regressors. For example, the efficient market hypothesis is inconsistent with the presence of a deterministic trend in an asset's return. Similarly, in testing for PPP we should not begin using a deterministic trend.

Third Step: A General to Specific Approach Dolado, Jenkinson and Sosvilla-Rivero (1990) suggest for following procedure to test for a unit root when the form of data-generating process (DGP) is *completely unknown*. We will see this stepwise procedure in the next section.

IV. A General to Specific Approach

> **SSW Rule** If the data-generating process (DGP) contains any deterministic regressors (i.e., an intercept or a time trend) and the estimating equation contains these deterministic regressors, inference on all coefficients can be conducted using a t-test or an F-test (Sims, Stock and Watson (1990), quoted from Enders, 2010, p.267).

Step 0 You can follow two different options to decide Model (0).

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(1) First option is applying a *general-to-specific approach* by, for example, following Ng and Perron (1995).

a) Set an upper bound p_{max} for p. A useful rule of thumb for determining p_{max} suggested by Schwert (1989) is $p_{\max} = \text{integer}\left[12\left(T/100\right)^{1/4}\right]$

b) Estimate the following equation by OLS with $p=p_{max}$

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

c) If the absolute value of the t statistic for testing the significance of the last lagged difference is greater than 1.6[1] then set $p=p_{max}$ and proceed to Step 1. Otherwise (if it is

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[1] Gretl uses 1.645 for this purpose.

lower than 1.6), reduce the lag length by one and repeat the process. Determine the lag length and the Model (0).

d) Use statistic LM test to check for autocorrelation in Model (0). The model must not have autocorrelation. If there is AC start from a model with larger lags and proceed.

(2) Another method to select the lag order *p* is using an information criterion: you can use information criteria in order to decide Model (0).

a) Estimate the following equation by OLS

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

b) Use statistic AIC[2] (or MAIC if there is) to find the proper number of differenced terms to be included in the equation.

c) Use LM test to test for the autocorrelation. If there is autocorrelation choose the model with second lowest AIC (or MAIC) value and check for autocorrelation, and so on.

Step 1 Start with Model (0): the general model with drift and deterministic trend. Use t_{ct}^{*} statistic to test $H_0 : \delta = 0$ versus $H_A : \delta < 0$. Unit root tests have low power to reject the null hypothesis, hence if H_0 is rejected, there is no need to proceed: we conclude that Y_t has no unit root.

Step 2 If $H_0 : \delta = 0$ is not rejected in Step 1, it is necessary to determine whether trend is significant. The presence of an unnecessary trend may have reduced the power of the test. You can test for the significance of the trend term by testing the hypothesis $H_0 : a_2 = \delta = 0$ using ϕ_3 statistic. If we do not reject $H_0 : a_2 = \delta = 0$,

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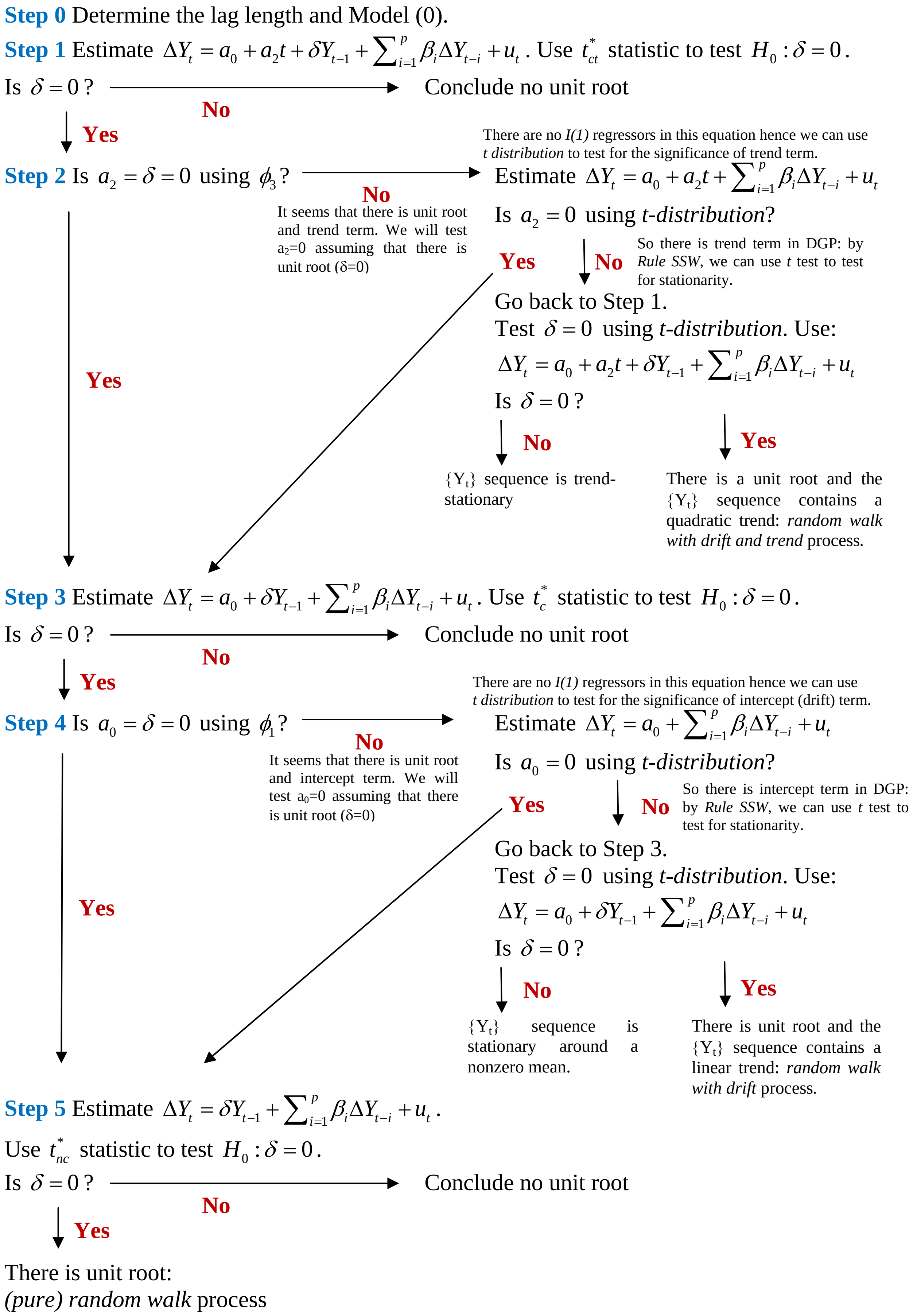
[2] Studies of the ADF statistic suggest that it is better to have too many lags then too few, so it is recommended to use AIC instead of SBC.

the the trend is not significant, proceed to Step 3. If the trend is significant (if we reject H_0) retest for the presence of unit root using the standard normal distribution. If the null of unit root is rejected, conclude that Y_t has no unit root. Otherwise Y_t has unit root is the conclusion.

Step 3 Estimate the model without trend. Test for the presence of a unit root using t_c^* statistic (or τ_c statistic). If the null hypothesis of a unit root is rejected, conclude that Y_t does not have a unit root. If the null hypothesis of a unit root is not rejected, test for the significance of drift (constant) by testing the hypothesis $H_0 : a_0 = \delta = 0$ using the ϕ_1 statistic. If the drift is not significant proceed to Step 4. If the drift is significant, test for the presence of a unit root $H_0 : \delta = 0$ using the standardized normal. If the null hypothesis of a unit root is rejected, conclude that Y_t has no unit root. Otherwise, conclude that Y_t has a unit root.

Step 4 Estimate the model without trend and drift. Test for the presence of a unit root using t_{nc}^* (or τ_{nc}) statistic. If the null hypothesis of a unit root is rejected, then conclude that Y_t does not contain a unit root. Otherwise, conclude that Y_t has a unit root.

Note that, this 4 step procedure cannot be expected well if it is used in a completely mechanical fashion. Plot the data and take into theoretical considerations.



Step 0 Determine the lag length and Model (0).

Step 1 Estimate $\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$. Use t_{ct}^{*} statistic to test $H_0 : \delta = 0$.

Is $\delta = 0$? → **No** → Conclude no unit root

↓ **Yes**

Step 2 Is $a_2 = \delta = 0$ using ϕ_3 ?

→ **No**

It seems that there is unit root and trend term. We will test a_2=0 assuming that there is unit root (δ=0)

There are no *I(1)* regressors in this equation hence we can use *t distribution* to test for the significance of trend term.

Estimate $\Delta Y_t = a_0 + a_2 t + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$

Is $a_2 = 0$ using *t-distribution*?

Yes → Step 3

↓ **No**

So there is trend term in DGP: by *Rule SSW*, we can use *t* test to test for stationarity.

Go back to Step 1.

Test $\delta = 0$ using *t-distribution*. Use:

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

Is $\delta = 0$?

↓ **No**

{Y$_t$} sequence is trend-stationary

↓ **Yes**

There is a unit root and the {Y$_t$} sequence contains a quadratic trend: *random walk with drift and trend* process.

↓ **Yes** (from Step 2)

Step 3 Estimate $\Delta Y_t = a_0 + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$. Use t_{c}^{*} statistic to test $H_0 : \delta = 0$.

Is $\delta = 0$? → **No** → Conclude no unit root

↓ **Yes**

Step 4 Is $a_0 = \delta = 0$ using ϕ_1?

→ **No**

It seems that there is unit root and intercept term. We will test a_0=0 assuming that there is unit root (δ=0)

There are no *I(1)* regressors in this equation hence we can use *t distribution* to test for the significance of intercept (drift) term.

Estimate $\Delta Y_t = a_0 + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$

Is $a_0 = 0$ using *t-distribution*?

Yes → Step 5

↓ **No**

So there is intercept term in DGP: by *Rule SSW*, we can use *t* to test for stationarity.

Go back to Step 3.

Test $\delta = 0$ using *t-distribution*. Use:

$$\Delta Y_t = a_0 + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

Is $\delta = 0$?

↓ **No**

{Y$_t$} sequence is stationary around a nonzero mean.

↓ **Yes**

There is unit root and the {Y$_t$} sequence contains a linear trend: *random walk with drift* process.

↓ **Yes** (from Step 4)

Step 5 Estimate $\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$.

Use t_{nc}^{*} statistic to test $H_0 : \delta = 0$.

Is $\delta = 0$? → **No** → Conclude no unit root

↓ **Yes**

There is unit root:
(pure) random walk process

V. Example

(1) Pure random walk

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(2) Random Walk with Drift

$$\Delta Y_t = a_0 + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

(3) Random Walk with Drift and Deterministic Trend

$$\Delta Y_t = a_0 + a_2 t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + u_t$$

t=1960-1995

Eq	a_0	δ	a_2	β_1	LM$_{AR(1)}$	F(ϕ3)	F(ϕ2)	F(ϕ1)
(3)	−0.1337 [−1.134]	−0.0117 [−0.582]	0.0183 [2.023]	0.9609 [5.453]	1.85 (0.276)	2.406	2.277	
(2)	0.0786 [1.398]	−0.0169 [−0.809]		1.1382 [7.099]	0.527 (0.468)			1.246
(1)		−0.0153 [−0.723]		1.1597 [7.161]	0.919 (0.345)			

Step 1

$H_0 : \delta = 0$
$H_A : \delta < 0$

$t_{\hat{\delta}}$=-0.582
t^*_{ct}=-3.50 for T=50 (There is no critical value for T=36, so take T=50)
$t_{\hat{\delta}}$ is not more negative than t^*_{ct}, $t_{\hat{\delta}} > t^*_{ct}$, hence, we do not reject the null hypothesis. Y_t has no unit root.

Step 2

We will test $H_0 : a_2 = \delta = 0$ versus $H_A : a_2 \neq 0 \; and \,/\, or \; \delta \neq 0$ using ϕ_3 statistic.

F(ϕ_3)=2.406
ϕ_3=6.73 (for T=50)
F(ϕ_3)<ϕ_3 so we do not reject the null hypothesis: the trend is not significant, we can proceed to Step 3.

Step 3

$H_0 : \delta = 0$
$H_A : \delta < 0$

$t_{\hat{\delta}}$=-0.809
t_c^*=-2.93 for T=50 (There is no critical value for T=36, so take T=50)
$t_{\hat{\delta}}$ is not more negative than t_c^*, $t_{\hat{\delta}} > t_c^*$, hence, we do not reject the null hypothesis. Y_t has no unit root.

Step 4

Since the null hypothesis of a unit root is <u>not rejected</u>, we test for the significance of drift (constant) by testing the hypothesis $H_0 : a_0 = \delta = 0$ using the ϕ_1 statistic.

$H_0 : a_0 = \delta = 0$
$H_A : a_0 \neq 0 \ and\ /\ or\ \ \delta \neq 0$

F(ϕ_1)=1.246
ϕ_1=4.86 (for T=50)
F(ϕ_1)<ϕ_1 so we do not reject the null hypothesis: the drift is not significant, we can proceed to Step 5.

Step 5

We test for the presence of a unit root using t_{nc}^* (or τ_{nc}) statistic.

$H_0 : \delta = 0$
$H_A : \delta < 0$

$t_{\hat{\delta}}$=-0.723
t^*_{nc}=-1.95 for T=50 (There is no critical value for T=36, so take T=50)
$t_{\hat{\delta}}$ is not more negative than t^*_{nc}, $t_{\hat{\delta}} > t^*_{nc}$, hence, we do not reject the null hypothesis. Y_t has no unit root.

Final Conclusion The time series Y_t is containing a unit root without drift and deterministic trend.

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