HANDOUT 06

TIME SERIES ANALYSIS IV: UNIT ROOT TEST UNDER STRUCTURAL CHANGE

Outline of this lecture:

I. Introduction

A stationary time-series may look like nonstationary when there are structural breaks in the intercept or trend. The unit root tests lead to false non-rejection of the null when we don't consider the structural breaks→ low power. A single breakpoint is introduced in Perron (1989) into the regression model; Perron (1997) extended it to a case of unknown breakpoint.

Perron (1989) argues that most macroeconomic variables are not unit root processes. They are Trend Stationary with Structural Breaks. For example: 1929 Depression, oil shocks, technological change. All these events have changed the mean of a process like GDP. If you do not take into account the structural break, you'll find unit root where there is not since when there is structural change all unit root tests are biased towards the non rejection of a unit root.

II. Effect of structural breaks in ADF test

We generate data $\{Y_t, t = 1,\ldots,100\}$ from the following process:

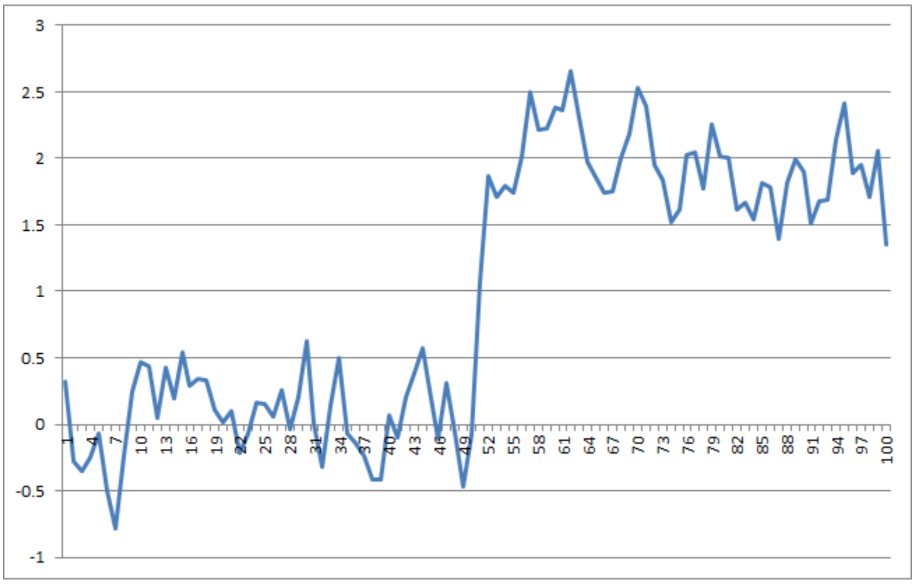
$$Y_t = 0.5Y_{t-1} + 3D_L + \varepsilon_t \qquad (1)$$

where

$$D_L = \begin{cases} 0 & t = 1,\ldots,50 \\ 1 & t = 51,\ldots 100 \end{cases}$$

$\varepsilon_t \sim WN(0,1)$, and $Y_0 = 0$.

Hence
$Y_t = 0.5Y_{t-1} + \varepsilon_t$ for t=1,…,50
$Y_t = 3 + 0.5Y_{t-1} + \varepsilon_t$ for t=51,…100.



The results from DF test applied to $\{Y_t\}$ are

$$\widehat{\Delta Y_t} = \underset{-0.98}{-0.0233}Y_{t-1}$$

$$\widehat{\Delta Y_t} = \underset{1.425}{0.0661} - \underset{-1.706}{0.0566} Y_{t-1}$$

$$\widehat{\Delta Y_t} = \underset{-0.689}{-0.0488} + \underset{2.118}{0.004} t - \underset{-2.734}{0.1522} Y_{t-1}$$

The comparison of the t-stat with the corresponding DF critical values leads to conclude that Y_t is an integrated process. This is a wrong conclusion since $\{Y_t\}$ is actually a stationary process in two sub-periods. This conclusion remains unchanged in many simulated data from the process (l). We can conclude that ADF test is biased under the presence of break in data. Some precaution must be taken when using ADF test from data with suspected structural changes. An alternative test is proposed by Perron (1989), Perron and Vogelsang (1992 and 1998), Zivot and Andrews (2002), Perron (1997), among a lot of others authors.

The endogenous structural break test of Zivot and Andrews (1992) is a sequential test which utilises the full sample and uses a different dummy variable for each possible break date. Here the break date is selected where the t-statistic from an ADF test of unit root is at a minimum (i.e. most negative). Consequently a break date will be chosen where the evidence is least favourable for the unit root null. The Zivot and Andrews (1992) minimum t-statistic has its own asymptotic theory and critical values. The latter are more negative than those provided by Perron (1989) and may suggest greater difficulty in rejecting the unit root null.

In addition to relaxing the assumption that breaks are known and discrete, further assumptions of Perron's (1989) initial paper have been examined in the literature. In particular the assumption of only one structural break has come under consideration and the possibility of multiple breaks is tested. Early work in this area includes papers by Lumsdaine and Papell (1997), Clemente et al. (1998) and Lee and Strazicich (2003).

III. Modeling structural changes

Three dummy variables are often used in structural change models. Let us denote the date of structural break as t_b. For example if the break occurs in period 51 then $t_b = 50$.

Some useful definitions can be summarized as follows:

- Sample period: $t=1,\dots,T$
- Break date (or date of change): t_b
- Pre-break sample: $t=1,\dots,t_b$
- Post-break sample: $t=t_b+1, \dots, T$

Single pulse dummy variable

Change becomes effective in t_b+1

$$DP_t = \begin{cases} 1 & \text{if } t = t_b + 1 \\ 0 & \text{if } t \neq t_b + 1 \end{cases}$$

Level dummy variable

Change becomes effective in t_b+1

$$DL_t = \begin{cases} 1 & \text{if } t > t_b \\ 0 & \text{if } t \leq t_b \end{cases}$$

Trend dummy variable

Change becomes effective in t_b+1

$$DT_t = \begin{cases} trend\,(= t - t_b) & \text{if } t > t_b \\ 0 & \text{if } t \leq t_b \end{cases}$$

IV. Perron's test of structural changes

Given a known structural break, Perron (1989, Econometrica) has proposed a modified DF test. His approach allows the following scenario tests:

1. Test for one change in level (intercept): model (A)
   - ✓ null hypothesis of a one-time jump in the level of a unit root process against the alternative of a one-time change in the intercept of a trend-stationary process.
2. Test for change in slope: model (B)
   - ✓ null hypothesis of a permanent change in the drift term of a random walk process versus the alternative of a change in the slope of the trend of a trend stationary process.
3. Test for changes in both level and slope: model (C)
   - ✓ structural change both in the level and drift of a unit root process versus a trend stationary process with a one-time change in the intercept and a change in the slope of the trend.

1. Test for one change in level (intercept): model (A)

Perron (1989) went on to develop a formal procedure to test for unit roots in the presence of a structural change at time t_b. Consider the *null hypothesis of a one-time jump in the level of a unit root process* against *the alternative of a one-time change in the intercept of a trend-stationary process*.

$$H_0^1 : Y_t = a_0 + Y_{t-1} + \mu_1 DP_t + \varepsilon_t$$
$$H_A^1 : Y_t = a_0 + a_2 t + \mu_2 DL_t + \varepsilon_t$$

An example of the DGP given in H_0^1 (Y_t is <u>*non-stationary*</u> with a structural break):

$$Y_t = 0.5 + Y_{t-1} + 20DP_t + \varepsilon_t ,$$

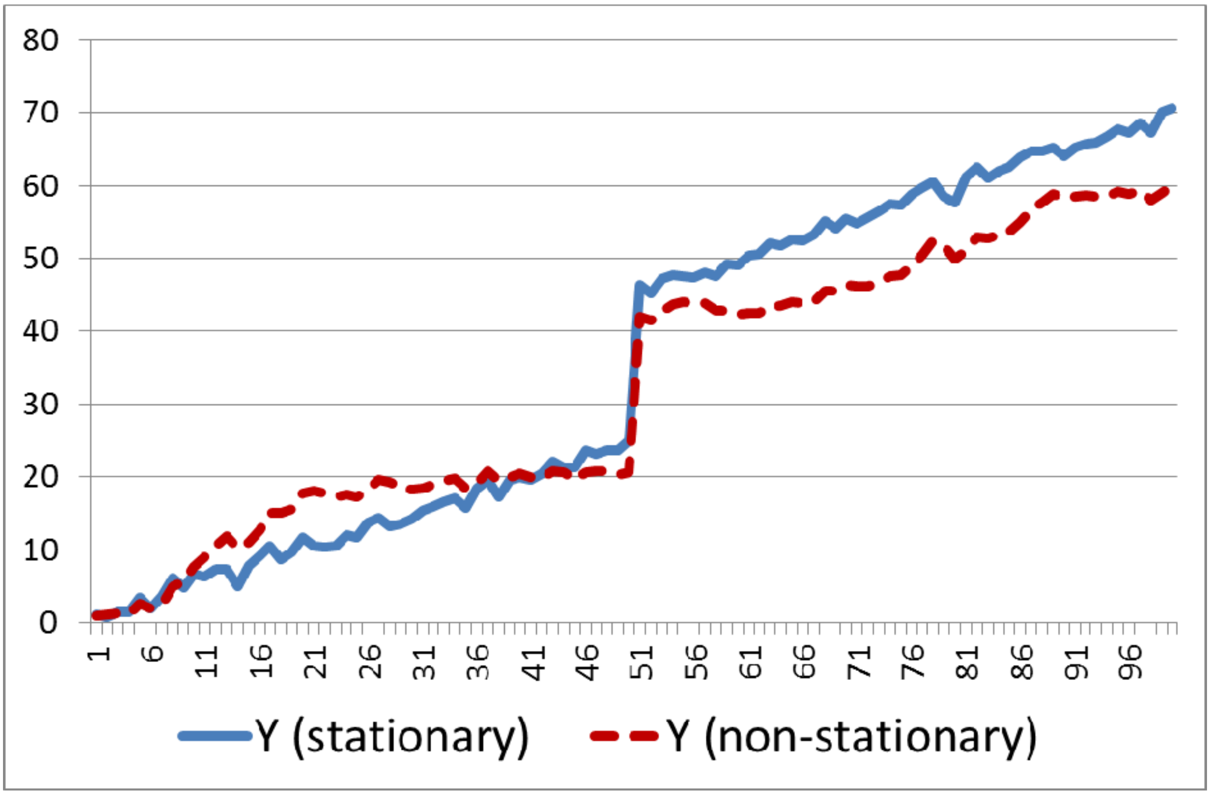
where $Y_0 = 0$, $DP_t = \begin{cases} 1 & \text{if } t = 51 \\ 0 & \text{if } t \neq 51 \end{cases}$

An example of the DGP given in H_A^1 (Y_t is *stationary* with a structural break):

$$Y_t = 0.1 + 0.5t + 20DL_t + \varepsilon_t$$

where $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

In the graph you see the plots of these two processes under alternative and null hypotheses.



2. Test for change in slope: model (B)

We can also test the *null hypothesis of a permanent change in the drift term of a random walk process* versus *the alternative of a change in the slope of the trend of a trend stationary process.*

$$H_0^2 : Y_t = a_0 + Y_{t-1} + \mu_2 DL_t + \varepsilon_t$$
$$H_A^2 : Y_t = a_0 + a_2 t + \mu_3 DT_t + \varepsilon_t$$

An example of the DGP given in H_A^2 (Y_t is <u>*stationary*</u> with a structural break):

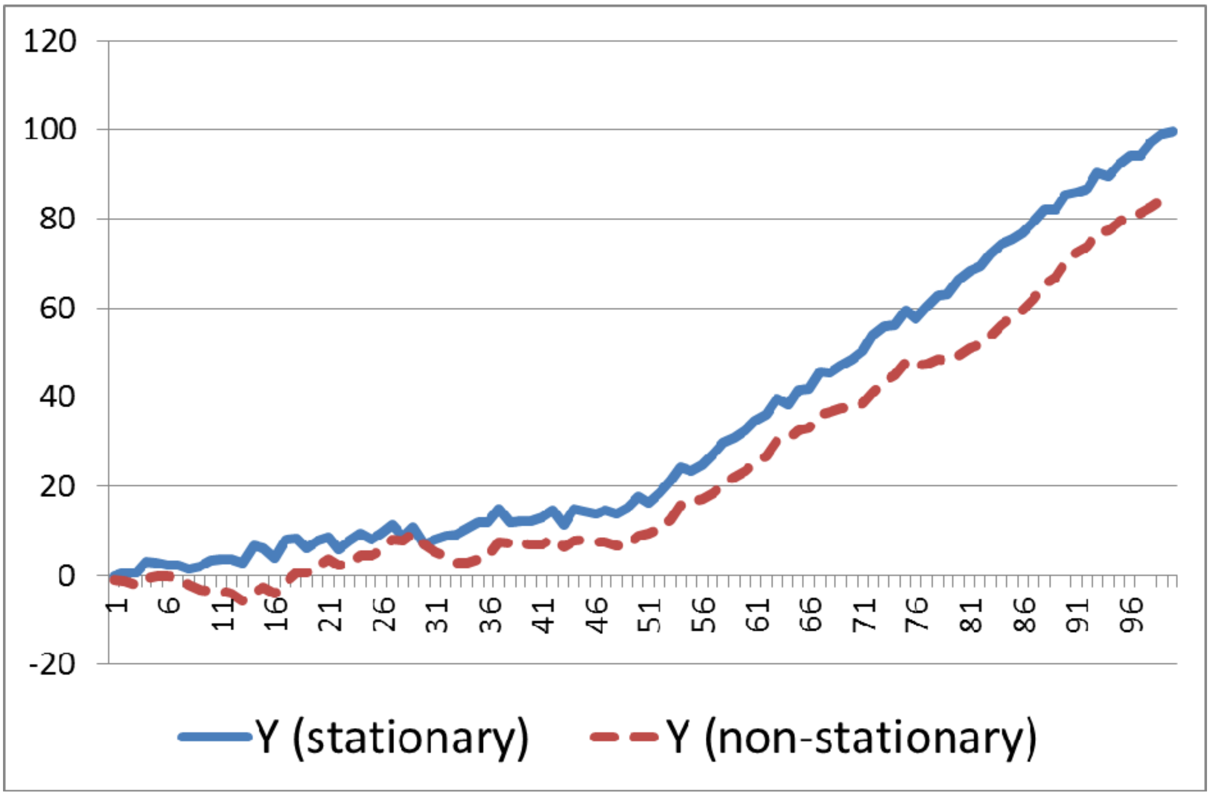
$$Y_t = 0.8 + 0.3t + 1.4DT_t + \varepsilon_t$$

where $DT_t = \begin{cases} trend\,(= t - 50) & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

An example of the DGP given in H_0^2 (Y_t is <u>*non-stationary*</u> with a structural break):

$Y_t = 0.3 + Y_{t-1} + 1.4DL_t + \varepsilon_t$ and $Y_0 = 0$, $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

In the graph below you see the plots of these two processes under alternative and null hypotheses.



3. Test for changes in both level and slope: model (C)

To be even more general, it is possible to combine the two null hypothesis H_0^1 and H_0^2. In this more general form a change in both the level and drift of a unit root process versus a trend stationary process (*a trend-stationary process with a one-time change in the intercept and a change in the slope of the trend*) can be tested.

$$H_0^3 : Y_t = a_0 + Y_{t-1} + \mu_1 DP_t + \mu_2 DL_t + \varepsilon_t$$
$$H_A^3 : Y_t = a_0 + a_2 t + \mu_2 DL_t + \mu_3 DT_t + \varepsilon_t$$

An example of the DGP given in H_0^3 (Y_t is <u>*non-stationary*</u> with a structural break):

$$Y_t = 0.5 + Y_{t-1} + 25D_P + 1.4D_L + \varepsilon_t$$

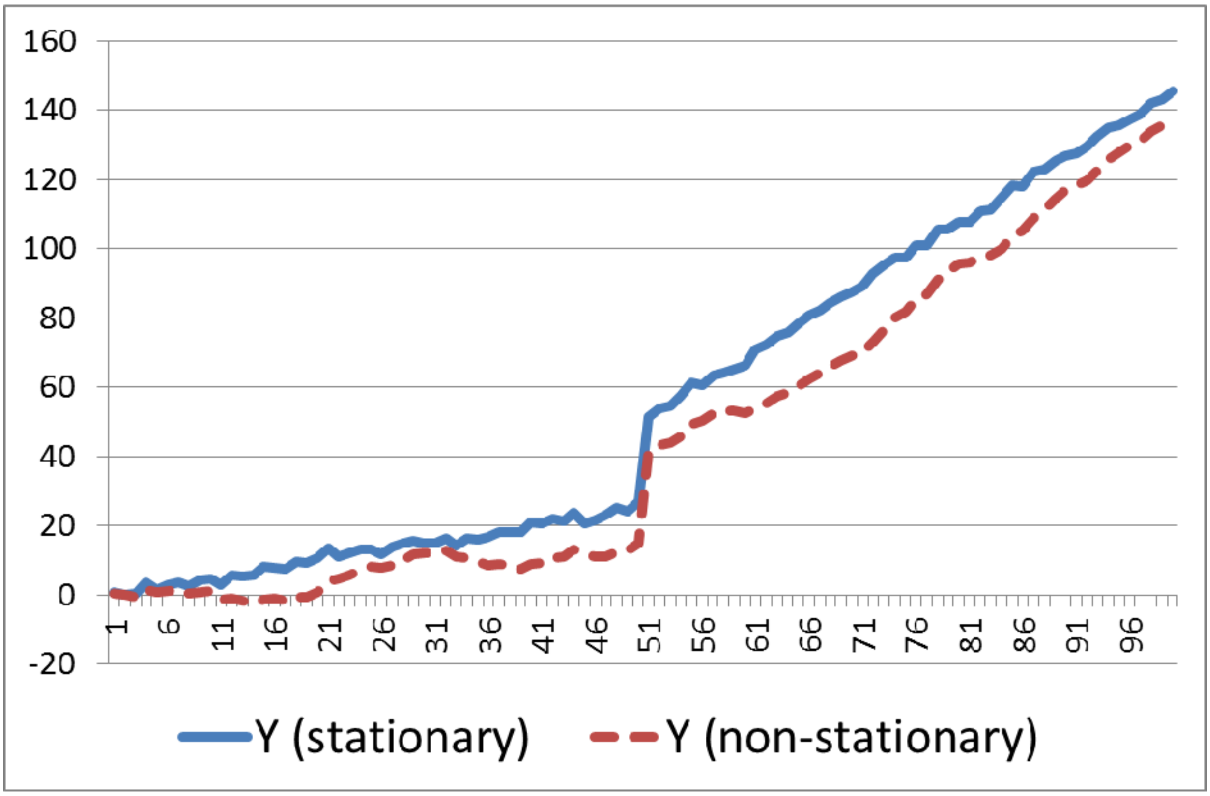
where $Y_0 = 0$, $DP_t = \begin{cases} 1 & \text{if } t = 51 \\ 0 & \text{if } t \neq 51 \end{cases}$ and $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

An example of the DGP given in H_A^3 (Y_t is <u>*stationary*</u> with a structural break):

$$Y_t = 0.1 + 0.5t + 25D_L + 1.4D_T + \varepsilon_t$$

where $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$ and $DT_t = \begin{cases} trend\,(= t - 50) & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

In the graph you see the plots of these two processes under alternative and null hypotheses.



4. Implementation of Perron's test

To conduct an *additive outlier* test Perron (1989) proposes the following steps:

Step 1 Detrend the data by estimating the model under the *alternative* hypothesis in A, B or C and calling the residuals as $\tilde{Y}_t$. Hence, $\tilde{Y}_t$ is the residual from the estimation of A, B or C equations.

Step 2 Estimate the regression:

$$\Delta\tilde{Y}_t = \delta\tilde{Y}_{t-1} + u_t$$

Under the null hypothesis of a unit root, the theorical value of $\delta = 0$ where $\delta = \rho - 1$. When the residuals u_t are *iid*, Perron (1989) showed that the distribution of δ and its *t* statistic depend on the proportion of observations occuring prior the break. Denote this proportion by $\lambda = t_b / T$.

Step 3 Check if the residuals from regression of step 2 are serially uncorrelated. If there is serial correlation, use the augmented form of the regression:

$$\Delta\tilde{Y}_t = \delta\tilde{Y}_{t-1} + \sum_{i=1}^{p}\beta_i\Delta\tilde{Y}_{t-i} + u_t$$

Step 4 Compare the t statistic for the null hypothesis $\delta = 0$ where $\delta = \rho - 1$. This statistic can be compared to the critical values calculated by Perron. Critical values depend on the proportion λ ranging from 0 to 1 by increments of 0.1. If you find a t-statistic more negative than the critical value calculated by Perron, it is possible to reject the null hypothesis of a unit root.

Table 1 Perron's critical values at 5%

	$\lambda = t_b / T$								
Models	***0.1***	***0.2***	***0.3***	***0.4***	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***
Crash Model: (A)	-3.68	3.77	3.76	-3.72	-3.76	-3.76	-3.8	-3.75	-3.69
Changing Growth Model: (B)	-3.65	-3.8	-3.87	-3.94	-3.96	-3.95	-3.85	-3.82	-3.68
Combined Model: (C)	-3.75	-3.99	-4.17	-4.22	-4.24	-4.24	-4.18	-4.04	-3.80

Source: Perron (1989)

In Perron's test (1989), the date of break is assumed to be known. In the case where this date is uncertain, you should consult Perron and Vogelsang (1992 and 1998), Zivot and Andrews (2002), Perron (1997). For a double breaks in mean see for exemple the Clemente Montanes and Reyes (1998).

5. Examples

Example 1 Suppose that $\hat{Y}_t$ has the DGP given in H_A^1 (i.e. Y_t is <u>*stationary*</u> with a structural break at *t*=50):

$$Y_t = 0.1 + 0.5t + 20DL_t + \varepsilon_t$$

where $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \le 50 \end{cases}$

You can find the data of this process in online.metu.edu.tr with the name of *a_h1.xls*

A standard ADF regression without any structural breaks gave:

$$\widehat{\Delta Y_t} = \underset{(0.1978)}{0.125} \underset{(-2.033)}{-1.01} Y_{t-1} + \underset{(1.988)}{0.0815} t - \underset{(-0.7894)}{0.0833} \Delta Y_{t-1} + \underset{(0.0813)}{0.0085} \Delta Y_{t-2}$$

$LM_{AR(1)}$=0.04196 [0.838] $LM_{AR(2)}$= 0.1098 [0.947]

The values in the brackets are corresponding probability values.

In the standard ADF regression $t_{\hat{\delta}}$ is -2.033 compared to a 5% critical value of -3.45. Hence the result indicates the *non-rejection* of the null hypothesis of a unit root since the *t* value is not more negative than the ADF critical value of -3.45. When we ignore the structural change, although the series is stationary, ADF test mistakenly points out non-stationarity.

$$\hat{Y}_t = \underset{(1.18)}{0.23} + \underset{(81.22)}{0.49} t + \underset{(58.41)}{20.4} DL_t \qquad (1)$$

$$\widehat{\Delta \tilde{Y}_t} = \underset{(-7.821)}{-1.096} \tilde{Y}_{t-1} + \underset{(1.540)}{0.156} \Delta \tilde{Y}_{t-1} \qquad (2)$$

$LM_{AR(1)}$= 0.283 [0.595] $LM_{AR(2)}$= 0.317 [0.853]

where $\tilde{Y}$ is the residual from the regression given in Eq. (1) and $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$. Here, $\lambda = t_b / T = 50/100 = 0.5$

The 5% asymptotic critical value for $t_{\hat{\delta}}$ with $\lambda = 0.5$ is -3.76 where $t_{\hat{\delta}} = -7.821$ indicating the rejection of the null hypothesis of a unit root.

Example 2 Suppose that $\hat{Y}_t$ has the DGP given in H_A^2 (i.e. Y_t is <u>*stationary*</u> with a structural break at *t*=50):

$$Y_t = 0.8 + 0.3t + 1.4DT_t + \varepsilon_t$$

where $DT_t = \begin{cases} trend\,(=t-50) & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

You can find the data of this process in online.metu.edu.tr with the name of *b_h1.xls*

A standard ADF regression without any structural breaks gave:

$$\widehat{\Delta Y_t} = \underset{(-1.07)}{-0.46} + \underset{(2.75)}{0.05}\,t \underset{(-0.87)}{-0.01} Y_{t-1} - \underset{(-5.08)}{0.52}\,\Delta Y_{t-1} - \underset{(-1.35)}{0.15}\,\Delta Y_{t-2} + \underset{(1.93)}{0.20}\,\Delta Y_{t-3}$$

$LM_{AR(1)}$=1.34 [0.25] $LM_{AR(2)}$= 1.42 [0.49]

The values in the brackets are corresponding probability values.

In the standard ADF regression $t_{\hat{\delta}}$ is -0.87 compared to a 5% critical value of -3.45. Hence the result indicates the *non-rejection* of the null hypothesis of a unit root since the *t* value is not more negative than the ADF critical value of -3.45. ADF test mistakenly points out non-stationarity.

$$\hat{Y}_t = \underset{(1.80)}{0.57} + \underset{(32.95)}{0.302}\,t + \underset{(86.03)}{1.396}\,DT_t \qquad (1)$$

$$\widehat{\Delta \tilde{Y}_t} = \underset{(-4.701)}{-0.887}\,\tilde{Y}_{t-1} - \underset{(-0.470)}{0.080}\,\Delta\tilde{Y}_{t-1} - \underset{(-0.220)}{0.031}\,\Delta\tilde{Y}_{t-2} - \underset{(1.523)}{0.156}\,\Delta\tilde{Y}_{t-3} \qquad (2)$$

$LM_{AR(1)}$= 1.06 [0.303] $LM_{AR(2)}$= 1.211 [0.546] where $\tilde{Y}$ is the residual from the regression given in Eq. (1) and $DT_t = \begin{cases} trend\,(=t-50) & \text{if } t>50 \\ 0 & \text{if } t \le 50 \end{cases}$. Here, $\lambda = t_b / T = 50/100 = 0.5$

The 5% asymptotic critical value for $t_{\hat{\delta}}$ with $\lambda = 0.5$ is -3.76 where $t_{\hat{\delta}} = -4.701$ indicating the rejection of the null hypothesis of a unit root.

Example 3 Suppose that $\hat{Y}_t$ has the DGP given in H_A^1 (i.e. Y_t is <u>*stationary*</u> with a structural break at *t*=50):

$$Y_t = 0.1 + 0.5t + 25D_L + 1.4D_T + \varepsilon_t$$

where $DL_t = \begin{cases} 1 & \text{if } t>50 \\ 0 & \text{if } t \le 50 \end{cases}$ and $DT_t = \begin{cases} trend\,(=t-50) & \text{if } t>50 \\ 0 & \text{if } t \le 50 \end{cases}$

You can find the data of this process in online.metu.edu.tr with the name of *c_h1.xls*

A standard ADF regression without any structural breaks gave:

$$\widehat{\Delta Y_t} = \underset{(-1.24)}{-1.17} + \underset{(2.52)}{0.10}t \underset{(-2.03)}{-0.05}Y_{t-1} - \underset{(-0.33)}{0.03}\Delta Y_{t-1} - \underset{(-0.33)}{0.03}\Delta Y_{t-2} + \underset{(0.71)}{0.07}\Delta Y_{t-2}$$

$LM_{AR(1)}$=0.005 [0.943] $LM_{AR(2)}$= 0.006 [0.997]

The values in the brackets are corresponding probability values.

In the standard ADF regression $t_{\hat{\delta}}$ is -2.03 compared to a 5% critical value of -3.45. Hence the result indicates the *non-rejection* of the null hypothesis of a unit root since the *t* value is not more negative than the ADF critical value of -3.45. When we ignore the structural change ADF test mistakenly points out non-stationarity.

$$\hat{Y}_t = \underset{(-1.05)}{-0.32} + \underset{(49.14)}{0.51}\,t + \underset{(57.93)}{24.48}\,D_L + \underset{(96.57)}{1.41}\,D_T \qquad (1)$$

where $DL_t = \begin{cases} 1 & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$ and $DT_t = \begin{cases} trend\,(=t-50) & \text{if } t > 50 \\ 0 & \text{if } t \leq 50 \end{cases}$

$$\widehat{\Delta\tilde{Y}_t} = \underset{(-5.444)}{-0.732}\,\tilde{Y}_{t-1} - \underset{(-1.839)}{0.184}\,\Delta\tilde{Y}_{t-1} \qquad (2)$$

$LM_{AR(1)}$= 0.125 [0.724] $LM_{AR(2)}$= 3.120 [0.21]

where $\tilde{Y}$ is the residual from the regression given in Eq. (1). Here, $\lambda = t_b / T = 50/100 = 0.5$

The 5% asymptotic critical value for $t_{\hat{\delta}}$ with $\lambda = 0.5$ is -3.76 where $t_{\hat{\delta}} = -5.444$ indicating the rejection of the null hypothesis of a unit root.

V. Spurious Regression

Granger and Newbold (1974) discovered the existence of "*spurious regressions*" which can occur when the results of a regression appear to look good in terms of having a high R^2 and significant t-statistics, however, the regression has no meaning.

A spurious regression is often characterised by the following:

1. high R^2 values, and
2. significant t-statistics, and
3. low Durbin-Watson (DW) statistics (Granger and Newbold, 1974, suggest $R^2 > DW$ as a rule of thumb)

The regression results are misleading since they indicate an economic relationship exists between these variables when it may be that no economic relationship exists.

- Term introduced by Granger and Newbold (1974)
- Refers to a regression among a set of unrelated non-stationary, I(1) variables.
- With regressions among non-stationary variables, conventional tests of significance (such as t and F tests, and R^2) tend to indicate a statistically significant relationship even when one does not really exist.
- Except in the special case where the variables cointegrate, regressions involving non-stationary variables are likely to give seriously misleading results.

References

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