# HANDOUT 07

# COINTEGRATION

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## 1. Cointegration

As a general rule, nonstationary time-series variables should not be used in regression models, to avoid the problem of spurious regression.

However, there is an exception to this rule. If $Y_t$ and $X_t$ are nonstationary I(1) variables, then we expect any linear combination of them such as $u_t = Y_t - \beta_0 - \beta_1 X_t$ to be I(1) as well.

However, there is an important case when $u_t = Y_t - \beta_0 - \beta_1 X_t$ is stationary I(0) process. In this case, $Y_t$ and $X_t$ are said to be cointegrated.

Cointegration implies that $Y_t$ and $X_t$ are share similar stochastic trends and since the difference $u_t$ is stationary they never diverge too far from each other. The cointegration is the statistical equilavent of the existence of a long-run economic relationship between I(1) variables.

A natural way to test whether $Y_t$ and $X_t$ are cointegrated is to test whether the disturbances $u_t = Y_t - \beta_0 - \beta_1 X_t$ are stationary.

Since we cannot observe $u_t$, we test the stationarity of the least squares residuals.

$$\hat{u}_t = Y_t - \hat{\beta}_0 - \hat{\beta}_1 X_t$$

using a Dickey-Fuller Test.

Hence, the test for cointegration is effectively a test of the stationary of the residuals.

- If the residuals are stationary then $Y_t$ and $X_t$ are said to be cointegrated.
- If the residuals are nonstationary, then $Y_t$ and $X_t$ are not cointegrated, any regression between them is said to be spurious.

The test for stationary of the residuals is based on the test equation:

$$\Delta\hat{u}_t = \gamma\hat{u}_{t-1} + v_t$$

where $\Delta\hat{u}_t = \hat{u}_t - \hat{u}_{t-1}$

As before, we examine the *t* (or tau) statistic for the estimated slope coefficient. Note that the regression has no constant term since the mean of the regression residual is zero.

The critical values for a cointegration are given below:

| **Models** | | **%1** | **%5** | **%10** |
|---|---|---|---|---|
| $\tau_{nc}$ | (1) $Y_t = \beta_1 X_t + u_t$ | -3.39 | -2.76 | -2.45 |
| $\tau_c$ | (2) $Y_t = \beta_0 + \beta_1 X_t + u_t$ | -3.96 | -3.37 | -3.07 |
| $\tau_{ct}$ | (3) $Y_t = \beta_0 + \alpha t + \beta_1 X_t + u_t$ | -3.98 | -3.42 | -3.13 |

The test equation can also include extra terms like $\Delta\hat{u}_{t-1}, \Delta\hat{u}_{t-2}\ldots$on the right hand side if they are needed to eliminate autocorrelation in $v_t$.

There are 3 sets of critical values as can be seen.

**Example**

You are given the following regression results and information:

$$\hat{\beta}_t = \underset{t\rightarrow(6.55)}{1.140} + \underset{(29.42)}{0.914} F_t \qquad R^2 = 0.881$$

where $\beta_t \sim I(1)$ and $F_t \sim I(1)$. The unit root test for stationary in the estimated residuals $\hat{u}_t$ is:

$$\Delta\hat{u}_t = \underset{(-4.196)}{-0.225}\hat{u}_{t-1} + 0.254\Delta\hat{u}_{t-1}$$

Note that this is the ADF version of the test with one lagged term $\Delta u_{t-1}$ to correct for autocorrelation. Since there is a constant term in the equation we use $t_c^*$

*$H_0$: the series are not cointegrated* [$\Leftrightarrow$ residuals are nonstationary]
*$H_A$: the series are cointegrated* [$\Leftrightarrow$ residuals are stationary]

Similar to the <u>one-tail</u> unit root tests, we reject $H_0$ of no cointegration if the *t*-values are more negative than the critical tau values.

The $t$ statistic ($t_{\hat{\gamma}}$) in this case is -4.196. Since this is more negative than the critical value -3.37, we reject the null hypothesis of no cointegration between $\beta_t$ and $F_t$. Hence $\beta_t$ and $F_t$ are cointegrated.

## 2. Error Correction Models

For example, suppose that the relationship between two I(1) variables, $Y_t$ and $X_t$ is characterized by the ARDL(1,1) model:

(1) $Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$

In (long-run) equilibrum

$$\bar{Y} = \beta_0 + \beta_1 \bar{Y} + \beta_2 \bar{X} + \beta_3 \bar{X}$$

So:

$$\bar{Y} - \beta_1 \bar{Y} = \beta_0 + (\beta_2 + \beta_3)\bar{X}$$

$$\bar{Y}(1 - \beta_1) = \beta_0 + (\beta_2 + \beta_3)\bar{X}$$

$$\bar{Y} = \frac{\beta_0}{1-\beta_1} + (\frac{\beta_2 + \beta_3}{1-\beta_1})\bar{X}$$

and

$$Y_t = \frac{\beta_0}{1-\beta_1} + (\frac{\beta_2 + \beta_3}{1-\beta_1})X_t$$

is the cointegrating relationship.

The ARDL(1,1) relationship in (1) may be rewritten to incorporate this relationship:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_t + \beta_2 X_{t-1} + \varepsilon_t$$

Subtracting $Y_{t-1}$ from both sides

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 - 1)Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$$

and subtracting $\beta_2 X_{t-1}$ from right hand side and adding it back again

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 - 1)Y_{t-1} + \beta_2 X_t - \beta_2 X_{t-1} + \beta_2 X_{t-1} + \beta_3 X_{t-1} + \varepsilon_t$$

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 - 1)Y_{t-1} + \beta_2 (X_t - X_{t-1}) + (\beta_2 + \beta_3) X_{t-1} + \varepsilon_t$$

$$Y_t - Y_{t-1} = (\beta_1 - 1)\left[\frac{\beta_0}{\beta_1 - 1} + Y_{t-1} + \frac{\beta_2 + \beta_3}{\beta_1 - 1} X_{t-1}\right] + \beta_2 (X_t - X_{t-1}) + \varepsilon_t$$

Hence we obtain the error correction model:

$$\Delta Y_t = (\beta_1 - 1)\left[Y_{t-1} - \frac{\beta_0}{1-\beta_1} - \frac{\beta_2 + \beta_3}{1-\beta_1} X_{t-1}\right] + \beta_2 \Delta X_t + \varepsilon_t$$

The model states that the change in Y in any period will be governed by the change in $X$ and the difference between $Y_{t-1}$ and the value predicted by the cointegrated relationship. The latter term is denoted the error correction mechanism $\left[Y_{t-1} - \frac{\beta_0}{1-\beta_1} - \frac{\beta_2 + \beta_3}{1-\beta_1} X_{t-1}\right]$, the effect of the term being to reduce the discrepancy.

$$\Delta Y_t \rightarrow I(0)$$

$$\Delta X_t \rightarrow I(0)$$

$$Y_{t-1} - \frac{\beta_0}{1-\beta_1} - \frac{\beta_2 + \beta_3}{1-\beta_1} X_{t-1} \rightarrow I(0)$$

So this model can be estimated by usual OLS.

## 3. Cointegration and Formal Definition

Sometimes two or more series have the same stochastic trend in common (*common stochastic trend*). In this special case, referred to as *cointegration*, regression analyses can reveal long-run relationships among time series variables.

The formal definition of cointegration for two I(1) series is as follows (Engle and Granger,1987)

***Definition*** Suppose that $Y_t$ and $X_t$ are integrated of order one. If, for some coefficient $\theta$,

$$Y_t - \theta X_t$$

is integrated of order zero, then $Y_t$ and $X_t$ are said to be cointegrated and denoted by $X_t, Y_t \sim C(1,1)$. The coefficient $\theta$ is called the cointegrating coefficient and the vector $[1, -\theta]$ is called a cointegrating vector. The term $Y_t - \theta X_t$ is called the error correction term.

If $X_t$ and $Y_t$ are cointegrated, then they have the same, or common, stochastic trend. Computing the difference $Y_t - \theta X_t$ eliminates this common stochastic trend (Stock and Watson, p.692).

If the number of variables involved in the long run relationship increases, the situation becomes much more complicated.

Consider, for example, a three variable case

$$Y_t = \beta_1 X_{t1} + \beta_2 X_{t2} + u_t$$

In this case, it is possible for the variables to be integrated of different orders and for the error term to be stationary.

In economics a more common situation would be:

$$Y_t \sim I(1),\ X_{t1} \sim I(2), \text{ and } X_{t2} \sim I(2)$$

Despite the different orders of integration, the error term could still be stationary $[u_t \sim I(0)]$ provided that $\beta_1 X_{t1} + \beta_2 X_{t2} \sim I(1)$ with cointegrating vector $[\beta_1, \beta_2]$.

# 4. Autoregressive Distributed Lag (ARDL) Model

Consider two (stationary) variables $Y_t$ and $X_t$ and assume that it holds that:

(1) $Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$

As an illustration, we can think of $Y_t$ as "company sales" and $X_t$ as "advertising" both in month *t*.

If we assume that $\varepsilon_t$ is a *white noise* process, independent of $X_t$, $X_{t-1}$ … and $Y_{t-1}$, $Y_{t-2}$… the above relation is referred to as an *Autoregressive Distributed Lag Model* (ARDL Model). To estimate it consistently, we can simply use OLS estimation (but the conditions must be met).

The Eq. (1) describes the dynamic effects of a change in $X_t$ on current and future values of $Y_t$. Taking partial derivatives, we can obtain the immediate response as follows.

$$\frac{\partial Y_t}{\partial X_t} = \beta_2$$

This is referred as "*impact multiplier*". An increase in X with one unit has an immediate impact on Y of $\beta_2$ units.

Since Eq (1) is as follows:

(1) $Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$

We can also write

(1') $Y_{t+1} = \beta_0 + \beta_1 Y_t + \beta_2 X_{t+1} + \beta_3 X_t + \varepsilon_{t+1}$

Note that Eq(1) shows that $Y_t$ is a function of $Y_{t-1}$, $X_t$ and $X_{t-1}$ so we can denote it as:

$$Y_t(Y_{t-1} X_t, X_{t-1})$$

On the other hand Eq(1') shows that $Y_{t+1}$ is a function of $Y_t$, $X_{t+1}$ and $X_t$; so likewise, we can denote it as:

$$Y_{t+1}(Y_t, X_{t+1}, X_t)$$

To obtain the effect of a change in $X_t$ on $Y_{t+1}$ (i.e., the effect after one period) we need to take partial derivative of $Y_{t+1}(Y_t, X_{t+1}, X_t)$ with respect to $X_t$ as follows:

$$m_1 = \frac{\partial\left[Y_{t+1}(Y_t, X_{t+1}, X_t)\right]}{\partial X_t} = \underbrace{\frac{\partial Y_{t+1}}{\partial Y_t}}_{\beta_1} \underbrace{\frac{\partial Y_t}{\partial X_t}}_{\beta_2} + \underbrace{\frac{\partial Y_{t+1}}{\partial X_t}}_{\beta_3}$$

hence

$$m_1 = \frac{\partial\left[Y_{t+1}(Y_t, X_{t+1}, X_t)\right]}{\partial X_t} = \beta_1.\beta_2 + \beta_3$$ → effect after one period.

We can also write:

(1'') $Y_{t+2} = \beta_0 + \beta_1 Y_{t+1} + \beta_2 X_{t+2} + \beta_3 X_{t+1} + \varepsilon_{t+2}$

So $Y_{t+2}$ is a function of $Y_{t+1}$, $X_{t+2}$ and $X_{t+1}$: $Y_{t+2}(Y_{t+1}, X_{t+2}, X_{t+1})$

Let us now obtain the effect of a change in $X_t$ on $Y_{t+2}$ (effect after two periods) by taking partial derivative of $Y_{t+2}(Y_{t+1}, X_{t+2}, X_{t+1})$ with respect to $X_t$:

$$m_2 = \frac{\partial\left[Y_{t+2}(Y_{t+1}, X_{t+2}, X_{t+1})\right]}{\partial X_t} = \frac{\partial Y_{t+2}}{\partial Y_{t+1}}\left[\frac{\partial Y_{t+1}}{\partial Y_t}\frac{\partial Y_t}{\partial X_t} + \frac{\partial Y_{t+1}}{\partial X_t}\right]$$

$$m_2 = \frac{\partial\left[Y_{t+2}(Y_{t+1}, X_{t+2}, X_{t+1})\right]}{\partial X_t} = \beta_1.(\beta_1.\beta_2 + \beta_3) \Rightarrow \text{effect after two periods.}$$

*Summarizing:*

- immediate response: $\beta_2$
- effect after one period: $\beta_1.\beta_2 + \beta_3$
- effect after two periods: $\beta_1.(\beta_1.\beta_2 + \beta_3)$
- effect after three periods: $\beta_1^{\,2}.(\beta_1.\beta_2 + \beta_3)$

(the last three: interim multipliers)

Note that effect is decreasing at each period if $|\beta_1| < 1$. Hence stability condition is $|\beta_1| < 1$.

Generalizing we can write:

$$m_t = \beta_1^{t-1}\left[\beta_1.\beta_2 + \beta_3\right]$$

Using this stability condition we can easily determine the long-run effect of a unit change in $X_{t.}$ It is given by the long-run multiplier (or equilibrium multiplier):

$$m_T = \beta_2 + (\beta_1\beta_2 + \beta_3) + \beta_1(\beta_1\beta_2 + \beta_3) + \beta_1^{\,2}(\beta_1\beta_2 + \beta_3) + \ldots\ldots$$

$$m_T = \beta_2\underbrace{\left[1 + \beta_1 + \beta_1^{\,2} + \beta_1^{\,3}\ldots.\right]}_{\frac{1}{1-\beta_1}\text{since } |\beta_1<1|} + \beta_3\underbrace{\left[1 + \beta_1 + \beta_1^{\,2} + \ldots.\right]}_{\frac{1}{1-\beta_1}}$$

$$m_T = \frac{\beta_2 + \beta_3}{1 - \beta_1}$$ → long-run multiplier in ARDL(1,1)

This says that if $X_t$ increases with one unit, the expected cumulated increase in sales is given by $\frac{\beta_2 + \beta_3}{1 - \beta_1}$.

This can also be obtained as follows:

(1) $Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$

In long-run equilibrium $\underbrace{Y = Y^*}_{E(Y_t)}$ and $\underbrace{X = X^*}_{E(X_t)}$

So;

$$Y^* = \beta_0 + \beta_1 Y^* + \beta_2 X^* + \beta_3 X^*$$

$$(1 - \beta_1) Y^* = \beta_0 + (\beta_2 + \beta_3) X^*$$

$$\Rightarrow Y^* = \frac{\beta_0}{1 - \beta_1} + (\frac{\beta_2 + \beta_3}{1 - \beta_1}) X^*,$$

which is the long-run equilibrium relation, or equivalently:

$$\Rightarrow E(Y_t) = \frac{\beta_0}{1 - \beta_1} + (\frac{\beta_2 + \beta_3}{1 - \beta_1}) E(X_t)$$

Then, this long-run equilibrium implies the following cointegrating relationship:

$$Y_t = \frac{\beta_0}{1 - \beta_1} + (\frac{\beta_2 + \beta_3}{1 - \beta_1}) X_t \qquad (2)$$

When $Y$ takes its equilibrium value with respect to $X$, Equation (2) holds. However economic systems are rarely in equilibrium. When $Y$ takes a value different from its equilibrium value, the difference between left-hand and right-hand sides of (2), that is $Y_t - \frac{\beta_0}{1-\beta_1} - (\frac{\beta_2+\beta_3}{1-\beta_1})X_t$ measures the extent of disequilibrium between the two variables. This quantity is, in fact, known as a disequilibrium error. It will, of course, take a zero value when $X$ and Y are in equilibrium.

The ARDL(1,1) relationship given in Eq(1) may be rewritten to incorporate this disequilibrium error.

(1) $$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$$

Subtracting $Y_{t-1}$ from both sides yields

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 - 1)Y_{t-1} + \beta_2 X_t + \beta_3 X_{t-1} + \varepsilon_t$$

Adding and subtracting $\beta_2 X_{t-1}$ from right hand side produces:

$$\Delta Y_t = \beta_0 + (\beta_1 - 1)Y_{t-1} + \beta_2 X_t - \beta_2 X_{t-1} + \beta_2 X_{t-1} + \beta_3 X_{t-1} + \varepsilon_t$$

$$\Delta Y_t = \beta_0 + (\beta_1 - 1)Y_{t-1} + \beta_2 (X_t - X_{t-1}) + (\beta_2 + \beta_3) X_{t-1} + \varepsilon_t$$

$$\Delta Y_t = \beta_0 + (\beta_1 - 1)Y_{t-1} + (\beta_2 + \beta_3) X_{t-1} + \beta_2 \Delta X_t + \varepsilon_t$$

$$\Delta Y_t = (\beta_1 - 1)\left[\frac{\beta_0}{\beta_1 - 1} + Y_{t-1} + \frac{\beta_2 + \beta_3}{\beta_1 - 1} X_{t-1}\right] + \beta_2 \Delta X_t + \varepsilon_t$$

Here note that for stationarity the condition is $|\beta_1| < 1$ which implies that $(\beta_1 - 1)$ must be negative, so we can write:

(3) $\Delta Y_t = \beta_2 \Delta X_t + (\beta_1 - 1)\left[ Y_{t-1} - \frac{\beta_0}{1-\beta_1} - \frac{\beta_2 + \beta_3}{1-\beta_1} X_{t-1} \right] + \varepsilon_t$

where $\beta_2$ is short run adjustment coefficient (or impact multiplier) and $\left[ Y_{t-1} - \frac{\beta_0}{1-\beta_1} - \frac{\beta_2 + \beta_3}{1-\beta_1} X_{t-1} \right]$ is disequilibrium error at t-1

Note that Eq(3) is another way of writing Eq(1). Eq(3) states that the current change in Y depend on the change in X and on the extent of disequilibrium in the previous period. Hence Eq(3) allows for any previous disequilibrium in the levels of X and Y.

Eq(3) is therefore referred to as a first-order error correction model (ECM). This is first-order since Eq(1) includes only first lags for Y and X.

We can write the cointegrating regression which includes the related cointegrating relationship as follows:

$$Y_t = \frac{\beta_0}{1-\beta_1} + \left( \frac{\beta_2 + \beta_3}{1-\beta_1} \right) X_t + u_t$$

or

$$Y_t = \beta_0^* + \beta_1^* X_t + u_t$$

where the cointegrating relationship is $Y_t = \beta_0^* + \beta_1^* X_t + u_t$ and the corresponding cointegrating vector is $\left[ 1, -\beta_0^*, -\beta_1^* \right]$ since the cointegrating relationship can also be written as $Y_t - \beta_0^* - \beta_1^* X_t = 0$

Consequently the ECM obtained in Eq(3) can be rewritten as:

$$\Delta Y_t = \beta_2 \Delta X_t + (\beta_1 - 1)u_{t-1} + \varepsilon_t$$

or

$$\Delta Y_t = \alpha_1 \Delta X_t + \alpha_2 u_{t-1} + \varepsilon_t$$

where $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1 - 1$ and $\alpha_2$ is <u>expected to be negative</u> since for the stability of ARDL(1,1) model $|\beta_1| < 1$

Suppose that $\Delta X_t$ is zero and $u_{t-1}$ is positive. This means that $Y_{t-1}$ is too high to be in equilibrium, that is $Y_{t-1}$ is above its equilibrium value of $\frac{\beta_0}{1-\beta_1} + \frac{\beta_2 + \beta_3}{1-\beta_1} X_{t-1}$.

Since $\alpha_2$ is expected to be negative, the term $\alpha_2 u_{t-1}$ is negative and therefore $\Delta Y_t$ will be negative to restore the equilibrium. That is, if $Y_t$ is above its equilibrium value, it will start falling in the next period to correct the equilibrium error; hence the name ECM comes from this process.

Note that, the absolute value of $\alpha_2$ determines how quickly the equilibrium is restored. Hence the <u>speed of adjustment</u> to equilibrium is dependent on the magnitude of $|\alpha_2|$ which is equal to be $1-\beta_1$ since $|\beta_1| < 1$.

- If $|\alpha_2| = 1$ then %100 of the adjustment takes place within a given period, or the adjustment is instantaneous and full.
- If $|\alpha_2| = 0.5$ then %50 of the adjustment takes place in each period.
- If $|\alpha_2| = 0$ then there is no adjustment.

In practice, since we do not know $u_{t-1}$ we estimate $u_{t-1}$ by $u_{t-1} = Y_t - \hat{\beta}_0^* - \hat{\beta}_1^* X_t$.

***Example***

$$\Delta Y_t = 0.362 \Delta X_t - 0.141 \hat{u}_{t-1}$$

t→ (9.6753) (-3.8461)

Statistically, the ECM term, (which is $\alpha_2$) is significant, suggesting that $Y_t$ adjusts to $X_t$ with a lag; only about 14 percent of the discrepancy between long-term and short-term $Y_t$ is corrected within a quarter. The impact multiplier (short-run impact) is about 0.36

## 5. Testing For Cointegration: The Engle-Granger Approach

Consider the two following series $X_t$ and $Y_t$. Suppose that we want to test if X and Y are cointegrated.

**STEP 1** Test two variables for the order of integration

The DF and ADF tests can be applied to test the order of integration of X and Y series.

a) If both are stationary $[I(0)]$, it is not necessary to proceed since in this case classical regression analysis can be applied.
b) If the variables are integrated of different order, it is possible to conclude that they are not cointegrated.
c) If both are integrated of the same order, we proceed with Step 2.

**STEP 2** Estimate the long run (cointegrating) relationship

If $X_t$ and $Y_t$ are integrated of the same order, the next step is to estimate the long-run equilibrium relationship.

$$Y_t = \beta_0^* + \beta_1^* X_t + u_t$$

and obtain the residuals $(\hat{u}_t)$ of this equation.

Keep mind that if there is no cointegration, the results will be spurious. However, if the variables are cointegrated, then OLS regression produces "*super-consistent*" estimators for the cointegrating parameter $\beta_1^*$.

**STEP 3** Check for (cointegration) the order of integration residuals.

We could use DF or ADF tests to test for the order of integration of $(\hat{u}_t)$.

The form of ADF test is:

$$\Delta\hat{u}_t = \delta\hat{u}_{t-1} + \sum_{i=1}^{p}\beta_i\Delta\hat{u}_{t-i} + v_t \qquad \text{Eq(**)}$$

Two things we should take into account in applying DF or ADF tests:

1) Eq(**) does not include a constant term (no drift) since by construction the OLS residuals $\hat{u}_t$ have zero mean.
2) The usual DF $t^*$ (tau or $\tau$) statistics are not appropriate for this test. Engle and Granger (1987), McKinnon (1991) and Davidson and McKinnon (1993) presented critical values fort his test. (this table will be given.)

$H_0 : \delta = 0$ (non stationarity of $u_t$; no cointegration)
$H_A : \delta < 0$ (stationarity of $u_t$; cointegration)

If the calculated values are more negative than the table values then we reject the null hypothesis. This implies that there is cointegration.

## **STEP 4** Estimate the ECM

If the variables are cointegrated, the residuals from the regression (cointegration regression) can be used to estimate the ECM and to analyze the LR and SR effects of the variables.

***Example***

You are given the following regression results where $C_t$ is private consumption and $Y_{t-1}$ is personal disposable income and t=1960,…,1995. Note that these equations are determined are according to a general-to-specific procedure and checked for AC by using LM tests.

(1) $$\Delta\hat{C}_t = \underset{(5.138)}{12330.48} - \underset{(-1.339)}{0.01091}C_{t-1}$$

$R^2$=0.052 $LM_{AR(1)}$=1.18

(2) $$\Delta^2\hat{C}_t = \underset{(4.301)}{7972.671} - \underset{(-4.862)}{0.85112}\Delta C_{t-1}$$

$R^2$=0.425 $LM_{AR(1)}$=0.94

(3) $$\Delta Y_t = \underset{(3.054)}{19903.9} - \underset{(-1.387)}{0.02479}Y_{t-1}$$

$R^2$=0.055 $LM_{AR(1)}$=1.24

(4) $$\Delta^2\hat{Y}_t = \underset{(3.983)}{12889.39} - \underset{(-6.27)}{1.11754}\Delta Y_{t-1}$$

$R^2$=0.551 $LM_{AR(1)}$=1.02

(5) $$\hat{C}_t = \underset{(3.123)}{11907.23} + \underset{(1.021)}{0.779585}Y_t$$

$R^2$=0.994 DW=1.021

(6) $$\Delta\hat{u}_t = -\underset{(-3.150)}{0.51739}\hat{u}_{t-1}$$

$R^2$=0.224 $LM_{AR(1)}$=2.98

(7) $$\Delta\hat{C}_t = \underset{(7.822)}{5951.557} + \underset{(6.538)}{0.28432}\Delta Y_t - \underset{(-2.486)}{0.1999}\hat{u}_{t-1}$$

$R^2$=0.572 DW=1.941 $LM_{AR(1)}$=0.007 [ $\rho$ =0.934]

| | $\tau_c^*$ | |
|---|---|---|
| *Sample size* | *1%* | *5%* |
| *25* | -3.75 | -3.00 |
| *50* | -3.58 | -2.93 |
| *100* | -3.51 | -2.89 |

## Solution

*Step 1*

$H_0 : \delta = 0$
$H_A : \delta < 0$

Eq (1) $t_{\hat{\delta}} = -1.339$
Do not $RH_0$ → there is unit root in $C_t$

Eq (2) $t_{\hat{\delta}} = -4.862$
$RH_0$ → $\Delta C_t$ stationary
Thus $C_t \sim I(1)$

Eq (3) $t_{\hat{\delta}} = -1.387$
Do not $RH_0$ → there is unit root in $Y_t$

Eq (4) $t_{\hat{\delta}} = -6.27$
$RH_0$ → $\Delta Y_t$ is stationary. Thus $Y_t \sim I(1)$

Hence, according to the ADF results: $C_t \sim I(1)$ and $Y_t \sim I(1)$.

*Step 2*

At this step, we have estimated Eq (5) and saved residuals, $\hat{u}_t$. The estimated cointegrating vector is $[1, -11907.23, -0.779585]$.

*Step 3*

Let us test the stationarity of $\hat{u}_t$. Using a general-to-specific procedure and checking for AC using LM we have obtained Eq (6). Now we will test the stationarity of $\hat{u}_t$ using the cointegration critical values of MacKinnon given below.

Critical Values for Two-Variable Cointegration ADF Test (Based on MacKinnon,1991)

| | *Level of Significance* | | |
|---|---|---|---|
| *Sample size* | *0.01* | *0.05* | *0.1* |
| *25* | -4.37 | -3.59 | -3.22 |
| *50* | -4.12 | -3.46 | -3.13 |
| *100* | -4.01 | -3.39 | -3.09 |
| ∞ | -3.90 | -3.33 | -3.05 |

$t_{\hat{\delta}} = -3.150 \Rightarrow$ If we test at $\alpha = 0.10$, the critical value -3.13. Then we reject $H_0$ of no cointegration.

However, if we work with a significance level less than 0.10, then those two variables are not cointegrated and we cannot say that there exists a long-run relationship between private consumption and personal disposable income.

*Step 4*

Using a general-to-specific procedure and checking for AC by LM, the following ECM given in Eq (7) is estimated. The results in Eq(7) show that short-run changes in personal disposable income $Y_t$ affect positively private consumption $C_t$.

Furthermore, because the short-run adjustment coefficient is significant $[\underset{(-2.486)}{-0.1999}]$, it shows that 0.1999 of the deviation of the actual private consumption from its long-run equilibrium level is corrected each year.

# References


- Dougherty, Christopher (2007) *Introduction to Econometrics*, Oxford, New York.
- Hill, R. C., Griffiths, W. E., and Judge, G. G., (2001) *Undergraduate Econometrics*, Second Edition, Wiley, New York.
- Kennedy, Peter (1998) *A Guide to Econometrics*, Fourth Edition, Blackwell, New york.
- Perron, P., (1989), "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," *Econometrica*, 57, 1361–1401.
- Petterson, K. (2000) *Applied Econometrics*, St. Martin's Press, New York.
- Stock, J., and Watson, M. M., (2012) *Introduction to Econometrics*, Third Edition, Pearson, Boston.