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Department of Economics
ECON 302

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PROBLEM SET 05 – TIME SERIES ANALYSIS

Problem 1

You are given the estimation results related to Y_t series for t=1947Q1-2008Q2 (246 observations). The values given in the brackets are probability values. The values within parenthesis are t values.

Test the stationarity of Y_t by applying the general-to-specific stepwise approach outlined in the ECON302 lecture.

(1)
$$\widehat{\Delta Y_t} = \underset{(3.32)}{0.03} + \underset{(1.99)}{0.0003}t - \underset{(-2.11)}{0.03}Y_{t-1} + \underset{(4.93)}{0.31}\Delta Y_{t-1} + \underset{(1.78)}{0.12}\Delta Y_{t-2} - \underset{(-0.78)}{0.05}\Delta Y_{t-3}$$
$$- \underset{(-0.53)}{0.03}\Delta Y_{t-4} - \underset{(-1.11)}{0.07}\Delta Y_{t-5} + \underset{(0.78)}{0.05}\Delta Y_{t-6} - \underset{(-0.88)}{0.06}\Delta Y_{t-7} - \underset{(-0.76)}{0.05}\Delta Y_{t-8}$$
$$+ \underset{(0.65)}{0.04}\Delta Y_{t-9} + \underset{(1.01)}{0.07}\Delta Y_{t-10} + \underset{(0.69)}{0.04}\Delta Y_{t-11} - \underset{(-3.00)}{0.19}\Delta Y_{t-12}$$

$LM_{AR(1)}$=0.95 [0.33] , $LM_{AR(2)}$= 1.77 [0.41], $LM_{AR(3)}$=1.95 [0.58] , $LM_{AR(4)}$= 2.93 [0.57]
$F(\phi_3)$=3.77, $F(\phi_2)$=10.65

(2)
$$\widehat{\Delta Y_t} = \underset{(4.64)}{0.01} - \underset{(-1.75)}{0.00001}t + \underset{(4.80)}{0.31}\Delta Y_{t-1} + \underset{(1.52)}{0.10}\Delta Y_{t-2} - \underset{(-1.04)}{0.07}\Delta Y_{t-3}$$
$$- \underset{(-0.79)}{0.05}\Delta Y_{t-4} - \underset{(-1.40)}{0.09}\Delta Y_{t-5} + \underset{(0.55)}{0.04}\Delta Y_{t-6} - \underset{(-1.14)}{0.08}\Delta Y_{t-7} - \underset{(-0.95)}{0.06}\Delta Y_{t-8}$$
$$+ \underset{(0.50)}{0.03}\Delta Y_{t-9} + \underset{(0.86)}{0.06}\Delta Y_{t-10} + \underset{(0.48)}{0.03}\Delta Y_{t-11} - \underset{(-3.33)}{0.21}\Delta Y_{t-12}$$

$LM_{AR(1)}$= 1.78 [0.18] , $LM_{AR(2)}$= 3.23 [0.20], $LM_{AR(3)}$=3.24 [0.36] , $LM_{AR(4)}$= 3.69 [0.45]

(3)
$$\widehat{\Delta Y_t} = \underset{(4.40)}{0.01} - \underset{(-1.88)}{0.002}Y_{t-1} + \underset{(4.80)}{0.31}\Delta Y_{t-1} + \underset{(1.52)}{0.10}\Delta Y_{t-2} - \underset{(-1.03)}{0.07}\Delta Y_{t-3}$$
$$- \underset{(-0.79)}{0.05}\Delta Y_{t-4} - \underset{(-1.40)}{0.09}\Delta Y_{t-5} + \underset{(0.56)}{0.04}\Delta Y_{t-6} - \underset{(-1.14)}{0.07}\Delta Y_{t-7} - \underset{(-0.95)}{0.06}\Delta Y_{t-8}$$
$$+ \underset{(0.50)}{0.03}\Delta Y_{t-9} + \underset{(0.86)}{0.06}\Delta Y_{t-10} + \underset{(0.48)}{0.03}\Delta Y_{t-11} - \underset{(-3.33)}{0.21}\Delta Y_{t-12}$$

$LM_{AR(1)}$= 1.72 [0.19] , $LM_{AR(2)}$= 3.12 [0.21], $LM_{AR(3)}$=3.13 [0.37] , $LM_{AR(4)}$= 3.61 [0.46]
$F(\phi_1)$=13.82

(4)
$$\widehat{\Delta Y_t} = \underset{(4.88)}{0.01} + \underset{(5.06)}{0.32}\Delta Y_{t-1} + \underset{(1.63)}{0.11}\Delta Y_{t-2} - \underset{(-0.96)}{0.06}\Delta Y_{t-3}$$
$$- \underset{(-0.66)}{0.04}\Delta Y_{t-4} - \underset{(-1.28)}{0.08}\Delta Y_{t-5} + \underset{(0.67)}{0.04}\Delta Y_{t-6} - \underset{(-1.04)}{0.07}\Delta Y_{t-7} - \underset{(-0.84)}{0.06}\Delta Y_{t-8}$$

$$+\underset{(0.61)}{0.04}\Delta Y_{t-9}+\underset{(0.96)}{0.06}\Delta Y_{t-10}+\underset{(0.56)}{0.04}\Delta Y_{t-11}-\underset{(-3.21)}{0.20}\Delta Y_{t-12}$$

$LM_{AR(1)}$= 1.91 [0.17] , $LM_{AR(2)}$= 3.50 [0.17], $LM_{AR(3)}$=3.50 [0.32] , $LM_{AR(4)}$= 3.87 [0.42]

(5)
$$\widehat{\Delta Y_t}=\underset{(2.77)}{0.002}Y_{t-1}+\underset{(5.98)}{0.38}\Delta Y_{t-1}+\underset{(2.12)}{0.14}\Delta Y_{t-2}-\underset{(-0.42)}{0.03}\Delta Y_{t-3}$$
$$-\underset{(-0.04)}{0.003}\Delta Y_{t-4}-\underset{(-0.70)}{0.05}\Delta Y_{t-5}+\underset{(1.26)}{0.08}\Delta Y_{t-6}-\underset{(-0.43)}{0.03}\Delta Y_{t-7}-\underset{(-0.23)}{0.02}\Delta Y_{t-8}$$
$$+\underset{(1.17)}{0.08}\Delta Y_{t-9}+\underset{(1.40)}{0.09}\Delta Y_{t-10}+\underset{(1.07)}{0.07}\Delta Y_{t-11}-\underset{(-2.38)}{0.15}\Delta Y_{t-12}$$

$LM_{AR(1)}$= 1.31 [0.25] , $LM_{AR(2)}$= 2.76 [0.25], $LM_{AR(3)}$=2.76 [0.43] , $LM_{AR(4)}$= 2.78 [0.60]

Problem 2

You are given the estimation results related to monthly Y_t series for t=1980M1-2000M10 (250 observations). The values given in the brackets are probability values. The values within parenthesis are *t* values.

Test the stationarity of Y_t by applying the general-to-specific stepwise approach outlined in the ECON302 lecture.

(1)
$$\widehat{\Delta Y_t}=\underset{(4.58)}{1.61}+\underset{(1.48)}{0.02}t-\underset{(-1.48)}{0.02}Y_{t-1}$$

$LM_{AR(1)}$=48.1 [0.000] , $LM_{AR(2)}$= 48.9 [0.000], $LM_{AR(4)}$=50.3 [0.000] , $LM_{AR(12)}$= 52.5 [0.000]
F(ϕ_3)=1.11, F(ϕ_2)=78.6

(2)
$$\widehat{\Delta Y_t}=\underset{(4.42)}{1.42}+\underset{(2.60)}{0.03}t-\underset{(-2.62)}{0.03}Y_{t-1}+\underset{(7.70)}{0.44}\Delta Y_{t-1}$$

$LM_{AR(1)}$=0.7 [0.413] , $LM_{AR(2)}$= 1.7 [0.423], $LM_{AR(4)}$=2.0 [0.734] , $LM_{AR(12)}$= 5.3 [0.946]
F(ϕ_3)=3.44, F(ϕ_2)=18.4

(3)
$$\widehat{\Delta Y_t}=\underset{(4.43)}{0.68}-\underset{(-0.18)}{0.0001}t+\underset{(7.36)}{0.43}\Delta Y_{t-1}$$

$LM_{AR(1)}$=0.1 [0.721] , $LM_{AR(2)}$= 0.4 [0.833], $LM_{AR(4)}$=0.9 [0.921] , $LM_{AR(12)}$= 3.0 [0.995]

(4)
$$\widehat{\Delta Y_t}=\underset{(7.51)}{1.14}+\underset{(0.05)}{0.0001}t$$

$LM_{AR(1)}$=45.1 [0.000] , $LM_{AR(2)}$= 45.4 [0.000], $LM_{AR(4)}$=45.8 [0.000] , $LM_{AR(12)}$= 47.4 [0.000]

(5)
$$\widehat{\Delta Y_t}=\underset{(6.66)}{1.15}-\underset{(-0.6)}{0.001}Y_{t-1}$$

$LM_{AR(1)}$= 45.0 [0.000] , $LM_{AR(2)}$=45.2 [0.000], $LM_{AR(4)}$=45.7 [0.000] , $LM_{AR(12)}$= 47.3 [0.000]
F(ϕ_1)=116.2

(6)
$$\widehat{\Delta Y_t}=\underset{(4.16)}{0.71}-\underset{(-0.38)}{0.0003}Y_{t-1}+\underset{(7.37)}{0.43}\Delta Y_{t-1}$$

$LM_{AR(1)}$= 0.1 [0.715] , $LM_{AR(2)}$=0.4 [0.827], $LM_{AR(4)}$=0.9 [0.925] , $LM_{AR(12)}$= 3.0 [0.996]
$F(\phi_1)$=23.6

(7) $$\widehat{\Delta Y_t} = \underset{(15.27)}{1.14}$$

$LM_{AR(1)}$= 45.1 [0.000] , $LM_{AR(2)}$=45.4 [0.000], $LM_{AR(4)}$=45.8 [0.000] , $LM_{AR(12)}$= 47.4 [0.000]

(8) $$\widehat{\Delta Y_t} = \underset{(6.87)}{0.66} + \underset{(7.38)}{0.43}\Delta Y_{t-1}$$

$LM_{AR(1)}$= 0.1 [0.722] , $LM_{AR(2)}$=0.4 [0.835], $LM_{AR(4)}$=0.9 [0.919] , $LM_{AR(12)}$= 3.0 [0.995]

(9) $$\widehat{\Delta Y_t} = \underset{(12.65)}{0.006}Y_{t-1}$$

$LM_{AR(1)}$= 64.8 [0.000] , $LM_{AR(2)}$=66.1 [0.000], $LM_{AR(4)}$=68.2 [0.000] , $LM_{AR(12)}$= 70.3 [0.000]

(10) $$\widehat{\Delta Y_t} = \underset{(5.30)}{0.003}Y_{t-1} + \underset{(9.30)}{0.52}\Delta Y_{t-1}$$

$LM_{AR(1)}$= 1.7 [0.189] , $LM_{AR(2)}$=3.8 [0.149], $LM_{AR(4)}$=4.6 [0.329] , $LM_{AR(12)}$= 9.0 [0.701]

Problem 3

You are given the estimation results related to monthly Y_t series for t=1990M1-2010M10 (250 observations). The values given in the brackets are probability values. The values within parenthesis are *t* values.

Test the stationarity of Y_t by applying the general-to-specific stepwise approach outlined in the ECON302 lecture.

(1) $$\widehat{\Delta Y_t} = \underset{(-0.75)}{-0.18} + \underset{(2.10)}{0.01}t - \underset{(-1.29)}{0.004}Y_{t-1} + \underset{(11.19)}{0.58}\Delta Y_{t-1}$$

$LM_{AR(1)}$=0.2 [0.692] , $LM_{AR(2)}$= 0.6 [0.757], $LM_{AR(3)}$=2.5 [0.653] , $LM_{AR(4)}$= 9.6 [0.652]
$F(\phi_3)$=7.78, $F(\phi_2)$=12.9

(2) $$\widehat{\Delta Y_t} = \underset{(0.33)}{0.05} + \underset{(3.72)}{0.004}t + \underset{(11.17)}{0.58}\Delta Y_{t-1}$$

$LM_{AR(1)}$= 0.0 [0.778] , $LM_{AR(2)}$= 0.3 [0.845], $LM_{AR(3)}$=2.7 [0.605] , $LM_{AR(4)}$= 10.4 [0.585]

(3) $$\widehat{\Delta Y_t} = \underset{(1.92)}{0.24} + \underset{(3.32)}{0.0003}Y_{t-1} + \underset{(11.34)}{0.59}\Delta Y_{t-1}$$

$LM_{AR(1)}$=0.11 [0.741] , $LM_{AR(2)}$= 0.4 [0.809], $LM_{AR(3)}$=2.8 [0.59] , $LM_{AR(4)}$= 10.7 [0.558]
$F(\phi_1)$=17.00

(4) $\widehat{\Delta Y_t} = \underset{(4.69)}{0.48} + \underset{(13.75)}{0.66}\Delta Y_{t-1}$

$LM_{AR(1)}$= 1.2 [0.271] , $LM_{AR(2)}$= 2.9 [0.24], $LM_{AR(3)}$=3.7 [0.45] , $LM_{AR(4)}$= 10.8 [0.547]

(5) $\widehat{\Delta Y_t} = \underset{(5.47)}{0.004} Y_{t-1} + \underset{(12.08)}{0.61}\Delta Y_{t-1}$

$LM_{AR(1)}$= 0.28 [0.599] , $LM_{AR(2)}$= 0.8 [0.660], $LM_{AR(3)}$=2.6 [0.626] , $LM_{AR(4)}$= 8.5 [0.748]

Problem 4

You are given the following estimation results for t=1920…2009. The values given in the brackets are probability values. The values within parenthesis are *t* values.

a) Test the stationarity of Y_t.
b) Test the stationarity of Y_t if you know that there is a structural break at 1960.

(1) $\widehat{\Delta Y_t} = \underset{(1.756)}{0.012} Y_{t-1}$

$LM_{AR(1)}$=1.61 [0.204] , $LM_{AR(2)}$= 1.72 [0.423], AIC=449.3, SBC=451.8

(2) $\widehat{\Delta Y_t} = \underset{(1.65)}{0.99} - \underset{(-0.524)}{0.007} Y_{t-1}$

$LM_{AR(1)}$=1.95 [0.162] , $LM_{AR(2)}$= 2.22 [0.329], AIC=448.6, SBC=453.6

(3) $\widehat{\Delta Y_t} = \underset{(-0.552)}{-0.454} + \underset{(2.506)}{0.115} t - \underset{(-2.562)}{0.138} Y_{t-1}$

$LM_{AR(1)}$=0.73 [0.393] , $LM_{AR(2)}$= 0.73 [0.694], AIC=444.3, SBC=451.8

Model (A) $\hat{Y}_t = \underset{(0.27)}{0.11} + \underset{(43.63)}{0.49} t + \underset{(36.21)}{21.07} DL_t$

$LM_{AR(1)}$=0.014 [0.905] , $LM_{AR(2)}$= 1.59 [0.452], AIC=318.5, SBC=325.96

Model (B) $\hat{Y}_t = \underset{(-4.14)}{-9.14} + \underset{(15.00)}{0.887} t - \underset{(-0.99)}{0.091} DT_t$

$LM_{AR(1)}$=65.97 [0.000] , $LM_{AR(2)}$= 66.12 [0.000], AIC=567.4, SBC=574.9

Model (C) $\hat{Y}_t = \underset{(1.92)}{1.17} + \underset{(24.14)}{0.45} t + \underset{(36.92)}{21.29} DL_t + \underset{(2.28)}{0.052} DT_t$

$LM_{AR(1)}$= 0.408 [0.523] , $LM_{AR(2)}$= 3.48 [0.176], AIC=315.2, SBC=325.2

(4) $$\widehat{\Delta\tilde{Y}_t^A} = \underset{(-9.454)}{-1.013}\tilde{Y}_{t-1}^A$$

LM$_{AR(1)}$= 2.58 [0.108] , LM$_{AR(2)}$= 3.62 [0.163], AIC=311.1, SBC=313.6

(5) $$\widehat{\Delta\tilde{Y}_t^B} = \underset{(-2.653)}{-0.143}\tilde{Y}_{t-1}^B$$

LM$_{AR(1)}$= 0.672 [0.412], LM$_{AR(2)}$= 0.673 [0.714], AIC=436.4, SBC=438.9

(6) $$\widehat{\Delta\tilde{Y}_t^C} = \underset{(-8.120)}{-1.277}\tilde{Y}_{t-1}^C + \underset{(1.75)}{0.188}\Delta\tilde{Y}_{t-1}^C$$

LM$_{AR(1)}$= 0.01 [0.919], LM$_{AR(2)}$= 0.07 [0.967], AIC=301.6, SBC=306.5

where $\tilde{Y}_t^A$ is the residual from the regression given in Model (A), $\tilde{Y}_t^B$ is the residual from the regression given in Model (B), $\tilde{Y}_t^C$ is the residual from the regression given in Model (C). Moreover: $DL_t = \begin{cases} 1 & \text{if } t > 1959 \\ 0 & \text{if } t \le 1959 \end{cases}$, and $DT_t = \begin{cases} t-1959 & \text{if } t > 1959 \\ 0 & \text{if } t \le 1959 \end{cases}$

Problem 5

You are given the following estimation results for t=1920…2009. The values given in the brackets are probability values. The values within parenthesis are *t* values.

a) Test the stationarity of Y_t.
b) Test the stationarity of Y_t if you know that there is a structural break at 1960.

(1) $$\widehat{\Delta Y_t} = \underset{(4.27)}{0.023}Y_{t-1}$$

LM$_{AR(1)}$=0.10 [0.756] , LM$_{AR(2)}$= 0.791 [0.673], AIC=469.5, SBC=471.99

(2) $$\widehat{\Delta Y_t} = \underset{(1.87)}{0.99} + \underset{(1.47)}{0.012}Y_{t-1}$$

LM$_{AR(1)}$=0.004 [0.952] , LM$_{AR(2)}$= 1.318 [0.517], AIC=468.0, SBC=472.99

(3) $$\widehat{\Delta Y_t} = \underset{(-1.14)}{-1.25} + \underset{(2.33)}{0.125}t \underset{(-1.878)}{-0.059}Y_{t-1}$$

LM$_{AR(1)}$=0.73 [0.393] , LM$_{AR(2)}$= 0.73 [0.694], AIC=464.6, SBC=472.03

Model (A) $\hat{Y}_t = \underset{(-9.41)}{-23.19} + \underset{(17.20)}{1.49}\,t + \underset{(2.49)}{11.24}\,DL_t$

LM$_{AR(1)}$=82.3 [0.000] , LM$_{AR(2)}$= 82.3 [0.000], AIC=687.3, SBC=694.8

Model (B) $\hat{Y}_t = \underset{(-3.69)}{-6.16} + \underset{(13.20)}{0.78}\,t + \underset{(16.33)}{1.51}\,DT_t$

LM$_{AR(1)}$=62.12 [0.000] , LM$_{AR(2)}$= 62.31 [0.000], AIC=567.3, SBC=574.8

Model (C) $\hat{Y}_t = \underset{(-1.06)}{-1.07} + \underset{(9.51)}{0.41}\,t + \underset{(13.76)}{18.20}\,DL_t + \underset{(30.98)}{1.63}\,DT_t$

LM$_{AR(1)}$= 58.53 [0.000] , LM$_{AR(2)}$= 58.53 [0.000], AIC=464.48, SBC= 474.48

(4) $\widehat{\Delta\tilde{Y}_t^A} = \underset{(-1.360)}{-0.033}\,\tilde{Y}_{t-1}^A$

LM$_{AR(1)}$= 0.42 [0.517] , LM$_{AR(2)}$= 0.78 [0.678], AIC=412.2, SBC=414.7

(5) $\widehat{\Delta\tilde{Y}_t^B} = \underset{(-2.879)}{-0.170}\,\tilde{Y}_{t-1}^B$

LM$_{AR(1)}$= 0.57 [0.449], LM$_{AR(2)}$= 0.848 [0.654], AIC=452.4, SBC=454.8

(6) $\widehat{\Delta\tilde{Y}_t^C} = \underset{(-2.984)}{-0.189}\,\tilde{Y}_{t-1}^C$

LM$_{AR(1)}$= 0.003 [0.955], LM$_{AR(2)}$= 1.20 [0.549], AIC=360.9, SBC=363.3

where $\tilde{Y}_t^A$ is the residual from the regression given in Model (A), $\tilde{Y}_t^B$ is the residual from the regression given in Model (B), $\tilde{Y}_t^C$ is the residual from the regression given in Model (C). Moreover, $DL_t = \begin{cases} 1 & \text{if } t > 1959 \\ 0 & \text{if } t \le 1959 \end{cases}$, and $DT_t = \begin{cases} t-1959 & \text{if } t > 1959 \\ 0 & \text{if } t \le 1959 \end{cases}$

Problem 6

You are given the following estimation results for t=1920…2009. The values given in the brackets are probability values. The values within parenthesis are *t* values.

a) Test the stationarity of Y_t.

b) Test the stationarity of Y_t if you know that there is a structural break at 1960.

(1)
$$\widehat{\Delta Y_t} = \underset{(2.00)}{0.04}\,Y_{t-1} - \underset{(-3.29)}{0.38}\,\Delta Y_{t-1} - \underset{(-3.59)}{0.44}\,\Delta Y_{t-2} + \underset{(0.83)}{0.11}\,\Delta Y_{t-3}$$
$$+ \underset{(0.51)}{0.07}\,\Delta Y_{t-4} + \underset{(0.95)}{0.13}\,\Delta Y_{t-5} + \underset{(1.41)}{0.19}\,\Delta Y_{t-6} - \underset{(-0.44)}{0.06}\,\Delta Y_{t-7} + \underset{(1.13)}{0.13}\,\Delta Y_{t-8}$$

$LM_{AR(1)}$=0.09 [0.765] , $LM_{AR(2)}$= 0.63 [0.73], AIC=387.2, SBC=408.7

(2)
$$\widehat{\Delta Y_t} = \underset{(1.42)}{0.67} + \underset{(1.97)}{0.03} Y_{t-1} - \underset{(-3.53)}{0.41} \Delta Y_{t-1} - \underset{(-3.85)}{0.48} \Delta Y_{t-2} + \underset{(0.35)}{0.05} \Delta Y_{t-3} + \underset{(0.07)}{0.01} \Delta Y_{t-4} + \underset{(0.50)}{0.07} \Delta Y_{t-5} + \underset{(0.97)}{0.13} \Delta Y_{t-6} - \underset{(-0.74)}{0.09} \Delta Y_{t-7} + \underset{(0.87)}{0.10} \Delta Y_{t-8}$$

$LM_{AR(1)}$=0.24 [0.630] , $LM_{AR(2)}$= 0.63 [0.730], AIC=386.9, SBC=410.9

(3)
$$\widehat{\Delta Y_t} = -\underset{(-1.68)}{2.31} + \underset{(2.30)}{0.08} t - \underset{(-1.18)}{0.04} Y_{t-1} - \underset{(-3.30)}{0.37} \Delta Y_{t-1} - \underset{(-3.83)}{0.46} \Delta Y_{t-2} + \underset{(0.43)}{0.06} \Delta Y_{t-3} + \underset{(0.25)}{0.03} \Delta Y_{t-4} + \underset{(0.74)}{0.10} \Delta Y_{t-5} + \underset{(1.30)}{0.18} \Delta Y_{t-6} - \underset{(-0.49)}{0.06} \Delta Y_{t-7} + \underset{(1.04)}{0.12} \Delta Y_{t-8}$$

$LM_{AR(1)}$=0.73 [0.393] , $LM_{AR(2)}$= 0.73 [0.694], AIC=383.0, SBC=409.4

Model (A)
$$\hat{Y}_t = \underset{(-12.01)}{-29.90} + \underset{(16.18)}{1.11} t - \underset{(-2.00)}{7.14} DL_t$$

$LM_{AR(1)}$=75.96 [0.000] , $LM_{AR(2)}$= 76.93 [0.000], AIC=645.4, SBC=652.9

Model (B)
$$\hat{Y}_t = \underset{(-2.96)}{-3.46} + \underset{(7.44)}{0.23} t + \underset{(26.22)}{1.28} DT_t$$

$LM_{AR(1)}$=20.5 [0.000] , $LM_{AR(2)}$= 21.3 [0.000], AIC=452.7, SBC=460.2

Model (C)
$$\hat{Y}_t = \underset{(-3.29)}{-4.30} + \underset{(6.68)}{0.27} t - \underset{(-1.40)}{1.73} DL_t + \underset{(25.74)}{1.27} DT_t$$

$LM_{AR(1)}$= 19.2 [0.000] , $LM_{AR(2)}$= 19.7 [0.000], AIC=452.7, SBC= 462.7

(4)
$$\widehat{\Delta \tilde{Y}_t^A} = \underset{(-1.76)}{-0.07} \tilde{Y}_{t-1}^A$$

$LM_{AR(1)}$=4.79 [0.029], $LM_{AR(2)}$= 13.15 [0.001], AIC=455.9, SBC=458.4

(5)
$$\widehat{\Delta \tilde{Y}_t^A} = \underset{(-1.815)}{-0.078} \tilde{Y}_{t-1}^A \underset{(-1.90)}{-0.22} \Delta \tilde{Y}_{t-1}^A \underset{(-2.51)}{-0.29} \Delta \tilde{Y}_{t-2}^A \underset{(1.67)}{+0.20} \Delta \tilde{Y}_{t-3}^A \underset{(1.26)}{+0.15} \Delta \tilde{Y}_{t-4}^A \underset{(1.51)}{+0.18} \Delta \tilde{Y}_{t-5}^A \underset{(2.07)}{+0.25} \Delta \tilde{Y}_{t-6}^A \underset{(-0.05)}{-0.01} \Delta \tilde{Y}_{t-7}^A \underset{(1.29)}{+0.15} \Delta \tilde{Y}_{t-8}^A$$

$LM_{AR(1)}$= 0.11 [0.744] , $LM_{AR(2)}$= 0.32 [0.853], AIC=405.7, SBC=427.2

(6)
$$\widehat{\Delta \tilde{Y}_t^B} = \underset{(-5.55)}{-0.52} \tilde{Y}_{t-1}^B$$

$LM_{AR(1)}$=1.80 [0.180], $LM_{AR(2)}$= 15.70 [0.001], AIC=419.7, SBC=422.2

(7) $$\widehat{\Delta\tilde{Y}_t^B} = \underset{(-2.42)}{-0.39}\tilde{Y}_{t-1}^B - \underset{(-1.04)}{0.17}\Delta\tilde{Y}_{t-1}^B - \underset{(-2.14)}{0.33}\Delta\tilde{Y}_{t-2}^B + \underset{(0.72)}{0.11}\Delta\tilde{Y}_{t-3}^B + \underset{(0.52)}{0.08}\Delta\tilde{Y}_{t-4}^B + \underset{(0.90)}{0.13}\Delta\tilde{Y}_{t-5}^B + \underset{(1.50)}{0.22}\Delta\tilde{Y}_{t-6}^B - \underset{(-0.22)}{0.03}\Delta\tilde{Y}_{t-7}^B + \underset{(1.22)}{0.14}\Delta\tilde{Y}_{t-8}^B$$

$LM_{AR(1)}$= 0.0003 [0.986], $LM_{AR(2)}$= 1.05 [0.591], AIC=369.2, SBC=390.8

(8) $$\widehat{\Delta\tilde{Y}_t^C} = \underset{(-5.67)}{-0.54}\tilde{Y}_{t-1}^C$$

$LM_{AR(1)}$=1.31 [0.253], $LM_{AR(2)}$= 14.20 [0.001], AIC=419.8, SBC=422.3

(9) $$\widehat{\Delta\tilde{Y}_t^C} = \underset{(-2.70)}{-0.45}\tilde{Y}_{t-1}^C - \underset{(-0.64)}{0.11}\Delta\tilde{Y}_{t-1}^C - \underset{(-1.76)}{0.27}\Delta\tilde{Y}_{t-2}^C + \underset{(1.00)}{0.15}\Delta\tilde{Y}_{t-3}^C + \underset{(0.77)}{0.12}\Delta\tilde{Y}_{t-4}^C + \underset{(1.11)}{0.16}\Delta\tilde{Y}_{t-5}^C + \underset{(1.67)}{0.24}\Delta\tilde{Y}_{t-6}^C - \underset{(-0.07)}{0.01}\Delta\tilde{Y}_{t-7}^C + \underset{(1.28)}{0.14}\Delta\tilde{Y}_{t-8}^C$$

$LM_{AR(1)}$= 0.003 [0.959], $LM_{AR(2)}$= 0.94 [0.624], AIC=371.3, SBC=392.8

where $\tilde{Y}_t^A$ is the residual from the regression given in Model (A), $\tilde{Y}_t^B$ is the residual from the regression given in Model (B), $\tilde{Y}_t^C$ is the residual from the regression given in Model (C). Moreover, $DL_t = \begin{cases} 1 & \text{if } t > 1959 \\ 0 & \text{if } t \le 1959 \end{cases}$, and $DT_t = \begin{cases} t-1959 & \text{if } t > 1959 \\ 0 & \text{if } t \le 1959 \end{cases}$