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Department of Economics
ECON 302

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PROBLEM SET 09 – SIMULTANEOUS EQUATIONS

Problem 1

From the model

$$Y_{1t} = \beta_{10} + \beta_{12}Y_{2t} + \gamma_{11}X_{1t} + u_{1t}$$
$$Y_{2t} = \beta_{20} + \beta_{21}Y_{1t} + \gamma_{22}X_{2t} + u_{2t}$$

the following reduced-form equations are obtained;

$$Y_{1t} = \Pi_{10} + \Pi_{11}X_{1t} + \Pi_{12}X_{2t} + w_t$$
$$Y_{2t} = \Pi_{20} + \Pi_{21}X_{1t} + \Pi_{22}X_{2t} + v_t$$

a) Are the structural equations identified?
b) What happens to identification if it is known *a priori* that $\gamma_{11} = 0$?
c) Suppose that the estimated reduced-form equations are as follows:

$$\hat{Y}_{1t} = 4 + 3X_{1t} + 8X_{2t}$$
$$\hat{Y}_{2t} = 2 + 6X_{1t} + 10X_{2t}$$

i. Obtain the values of the structural parameters β_{10}, β_{12}, γ_{11}, β_{20}, β_{21} and γ_{22}.
ii. How would you test the null hypothesis that $\gamma_{11} = 0$?

Problem 2

From the model

$$Y_{1t} = \beta_{10} + \beta_{12}Y_{2t} + \gamma_{11}X_{1t} + u_{1t}$$
$$Y_{2t} = \beta_{20} + \beta_{21}Y_{1t} + u_{2t}$$

The following estimated reduced-form equations are obtained:

$$\hat{Y}_{1t} = 4 + 8X_{1t}$$
$$\hat{Y}_{2t} = 2 + 12X_{1t}$$

a) Obtain the values of the structural parameters β_{10}, β_{12}, γ_{11}, β_{20}, and β_{21}.

b) How does the answer to (a) change if it is known a priori that (1) $\beta_{12} = 0$ and (2) $\beta_{10} = 0$?

Problem 3

Table below is a model in five equations with five endogenous variables Y and four exogenous variables X:

	Coefficients of the variables								
Equation no.	Y_1	Y_2	Y_3	Y_4	Y_5	X_1	X_2	X_3	X_4
1	1	β_{12}	0	β_{14}	0	γ_{11}	0	0	γ_{14}
2	0	1	β_{23}	β_{24}	0	0	γ_{22}	γ_{23}	0
3	β_{31}	0	1	β_{34}	β_{35}	0	0	γ_{33}	γ_{34}
4	0	β_{42}	0	1	0	γ_{41}	0	γ_{43}	0
5	β_{51}	0	0	β_{54}	1	0	γ_{52}	γ_{53}	0

Determine the identifiability of each equation with the aid of the order and rank conditions of identifications.

Problem 4

Consider the following demand-and-supply model for money;

$$M_t^d = \beta_0 + \beta_1 Y_t + \beta_2 R_t + \beta_3 P_t + u_t$$
$$M_t^s = \alpha_0 + \alpha_1 Y_t + u_{2t}$$

where M=money Y=income R=rate of interest P=price

a) Is the demand function identified?

b) Is the supply function identified?

c) Which method would you use to estimate the parameters of the identified equation(s)?

d) Suppose we modify supply function by adding explanatory variables Y_{t-1} & M_{t-1}.What happens to the identification problem? Would you still use the method you used in (c)?

Problem 5

Consider the following model:

$$R_t = \beta_0 + \beta_1 M_t + \beta_2 Y_t + u_{1t}$$
$$Y_t = \alpha_0 + \alpha_1 R_t + u_{2t}$$

where M_t (money supply) is exogenous, R_t is the interest rate, and Y_t is GDP.

a) Are the equations identified?

a) Which method would you use to estimate the parameters of the identified equation(s)?

Problem 6

Consider the following model

$$R_t = \beta_0 + \beta_1 M_t + \beta_2 Y_t + \beta_3 Y_{t-1} + u_{1t}$$
$$Y_t = \alpha_0 + \alpha_1 R_t + u_{2t}$$

b) Find out if the system is identified.

c) Which method would you use to estimate the parameters of the identified equation(s)?

Problem 7

You are given the following model;

$$R_t = \beta_0 + \beta_1 M_t + \beta_2 Y_t + u_{1t}$$
$$Y_t = \alpha_0 + \alpha_1 R_t + \alpha_2 I_t + u_{2t}$$

where the variables are as defined in the previous exercise. Treating I (domestic investment) and M exogenously, determine the identification of the system. Which method would you use to estimate the parameters of the identified equation(s)?