PROBLEM SET 09 – SIMULTANEOUS EQUATIONS - Solutions

Problem 1

(*a*) By the order condition, Y1 and *Y*2 are both exactly identified.

(*b*) In this case Y_1 is identified, but not Y_2.

(**c**)

$$\hat{\beta}_{10} = [\hat{\pi}_{20} - \frac{\hat{\pi}_{22}}{\hat{\pi}_{12}}\hat{\pi}_{10}] = -3; \hat{\beta}_{12} = \frac{\hat{\pi}_{22}}{\hat{\pi}_{12}} = 1.25$$

$$\hat{\beta}_{20} = [\hat{\pi}_{20} - \frac{\hat{\pi}_{21}}{\hat{\pi}_{11}}\hat{\pi}_{10}] = -6; \hat{\beta}_{21} = \frac{\hat{\pi}_{21}}{\hat{\pi}_{11}} = 2$$

$$\hat{\gamma}_{11} = [\hat{\pi}_{21} - \frac{\hat{\pi}_{11}\hat{\pi}_{21}}{\hat{\pi}_{11}}] = 2.25; \hat{\gamma}_{12} = [\hat{\pi}_{22} - \frac{\hat{\pi}_{12}\hat{\pi}_{21}}{\hat{\pi}_{11}}] = -6$$

(**d**) To test this hypothesis, we need the standard error of $\hat{\gamma}_{11}$. But as you can see from (*a*), $\hat{\gamma}_{11}$ is a nonlinear function of the $\hat{\pi}$ coefficients and it is not easy to estimate its standard error.

Problem 2

(*a*) In this example, Y_1 is not identified but Y_2 is.

$$\hat{\beta}_{21} = \frac{\hat{\pi}_3}{\hat{\pi}_1} = 1.5; \hat{\beta}_{20} = (\hat{\pi}_2 - \hat{\beta}_{21}\hat{\pi}_0) = -4$$

The other structural coefficients cannot be identified.

(*b*) In this case both Y_1 and Y_2 are identified.

Problem 3

By the order condition, $Y_1, Y_2,$ and Y_5 are just identified, Y_3 is not identified and Y_4 is overidentified.

By rank condition all equations are identified.

Problem 4

(*a*) The demand function is unidentified.

(*b*) The supply function is overidentified.

(*c*) 2SLS may be used to estimate the parameters of the overidentified supply function.

(*d*) Both the functions are now overidentified. Hence, use 2SLS.

Problem 5

(*a*) By the order condition, the interest rate equation is not identified, but the income equation is just identified.

(*b*) In this example, M is the exogenous variable. Using the data given in Table 20.2, we obtain the following results by ILS:

$$\hat{Y}_t = 2834.488 + 1.2392 M_t$$
$$t = (32.0163) \ (37.3812)$$
$$r^2 = 0.9803; d = 0.3074$$

It is left as an exercise for the reader to retrieve the original structural coefficients, namely, α_0 and α_1.

Problem 6

(*a*) By the order condition, the interest rate equation is not identified, and the income equation is overidentified.

(*b*) Here you may use 2SLS.

Problem 7

Here both the equations are exactly identified. One can use ILS or 2SLS to estimate the parameters, but they will give identical results for reasons discussed in the chapter.

Here are the OLS estimates of the reduced form (RF) equations. Note that in the RF, only the exogenous variables (I and M) appear on the right side of each equation.

$$\hat{R}_t = 8.7056 - 0.00049 M_t - 0.00084 I_{t-1}$$
$$t = (6.0589)(-05192) \quad (-0.2281)$$
$$R^2 = 0.1172$$
$$\hat{Y}_t = 2421.074 + 0.8944 M_t + 1.4585 I_{t-1}$$
$$t = (32.7247) \ (18.3144) \quad (7.6607)$$
$$R^2 = 0.9938$$

It is left for the reader to retrieve the original structural parameters from the reduced form coefficients.