Middle East Technical University **November 2013**
Department of Economics
ECON 302

PROBLEM SET III – AUTOCORRELATION

Problem 1

Consider the following model of demand for airline travel, estimated using annual data for the period 1947-1987. The number of observations is therefore 41.

$$\ln Q_t = \beta_1 + \beta_2 \ln P_t + \beta_3 \ln Y_t + \beta_4 \ln ACCID_t + \beta_5 \ln FATAL_t + u_t$$

Q=Per-capita passenger miles traveled in a given year.
P=Average price per mile
Y=Per capita income
ACCID=Accident rate per passenger mile
FATAL=Number of fatalities from aircraft accident

The model is double-log except for the fact that FATAL is not expressed in log from because the observation for some of the years is zero. The model was estimated by OLS and Durbin-Watson statistic is 0.97

a. Explain briefly the steps needed to obtain an LM test for first order autocorrelation.
b. For the Durbin-Watson test, write down the ranges of critical values for a 5 percent critical level of significance.
c. Carry out the test and state whether you find significant autocorrelation.

Problem 2

Assume that the disturbance term exhibits the first order autoregressive scheme, AR(1);

$u_t = \rho\, u_{t-1} + e_t$

where e_t satisfies all the assumptions of the classical linear regression model.

a. Show that

$$\text{Var}(u_t) = \frac{\sigma_e^2}{(1-\rho^2)} \quad \text{where } \sigma^2_e = \text{Var}(e_t)$$

b. What is the covariance between u_t and u_{t-1}? Between u_t and u_{t-2}? Generalize your results.
c. Write down the variance-covariance matrix of the u's.

Problem 3

Let C be real per-capita consumption in the US at time t, and Y be real per-capita disposable income, both measured in billions of dollars. Using the annual data for 32 years, the following model was obtained.

$$\hat{C}_t = -21151 + 2989.9 \ln Y_t \qquad \text{where } AdjR^2 = 0.985 \quad d = 0.207$$
$$(-39.3) \quad (45.0)$$

The values in parenthesis are t statistics.

a. Test the model for first-order serial correlation at the 5 % level of significance.
b. Based on your conclusion what can you say about properties of the OLS estimates just given, in terms of unbiasedness, BLUE and the validity of the tests of hypothesis.

It is hypothesized that C depends on the consumption in the previous period but is adjusted for changes in disposable income. The following model was obtained using OLS;

$$\hat{C}_t = -21.83 + 1.01\, C_{t-1} + 0.769\, (Y_t - Y_{t-1})$$
$$(-0.78) \quad (107.5) \qquad (7.8)$$
$$AdjR^2 = 0.998 \quad d = 2.11 \qquad \rho = -0.07$$

c. Are these estimates unbiased? Explain.
d. Test the model for first order autocorrelation. Write down the null and the alternative hypothesis.
e. Based on your results in part e, are the OLS estimates unbiased and consistent? Explain.

Problem 4

The following equations are estimated for the 1982.1-1985.12 period:

(1) $\hat{BT}_t = 20451.2 - 217.5073\, XR_t$

(4040.9) 31.4952)

R^2=0.5090 SSR=360743047.4 DW=0.6288

(2) $\hat{BT}_t$ = 19795.5 - 206.7189 XR_t - 46030.3 D_t + 302.5339 $XR_t D_t$
(3255.7) (25.1104) (11342.9) (82.0540)

R^2=0.7535 SSR=181087442.4 DW=1.3627

The figures in the parenthesis are standard errors and D=0 for 1982.04-1985.03 and 1 otherwise.

Test for the presence of first-order autocorrelation in (1) and (2) using the possible statistics. Interpret your results.

Problem 5

You are given time series data for the period 1977.1-1991.2 on the estimation of aggregate consumption by disposable income.

(OLS)(1)	C=13.2 + 0.88Y (3.38) (0.01)	R^2=0.988	DW=1.11
(OLS)(2)	C=5.08 + 0.64C_{-1} + 0.33Y (3.00) (0.10) (0.09)	R^2=0.993	DW=2.12
(EGLS)(3)	C=14.98 + 0.87Y (5.29) (0.02)	$\hat{p}$=0.45	DW=2.27

a. Test for autocorrelation in (1).

b. As a solution to the autocorrelation in (1) would you prefer (2) or (3) ? Explain carrying out the necessary tests.

Problem 6

Consider the following regression model:

$\hat{Y}_t$ = -49.4664 + 0.88544 X_{2t} + 0.09253 X_{3t}
t → (-2.2392) (70.2936) (2.6933)

R^2=0.9979, DW=0.8755

where,

Y=the personal consumption expenditure

X_2=the personal disposable income
X_3=the Dow Jones Industrial Average Stock Index

The regression is based on US data from 1961 to 1985.

a. Is there a first order autocorrelation in the residuals of this regression? How do you know?
b. The following model was estimated by transforming the model as;

$$(Y_t - \rho Y_{t-1})=\beta_0 (1 - \rho) + \beta_1 (X_{t2} - \rho X_{(t-1)2}) + \beta_2 (X_t - \rho X_{(t-1)3}) + u_t$$

$$Y=-17.97 + \underset{(30.72)}{0.89} X + \underset{(2.66)}{0.09} X \qquad R^2=0.9816, \; DW=2.28$$

Has the problem of autocorrelation been resolved? How do you know?

Problem 7

Consider the following model

$$Y_t=b_0 + b_1 X_t + u_t \qquad ; \qquad u_t=\rho u_{t-1} + e_t$$

and,

$E(e_t)=0$
$E(e_t^2)=\sigma_e^2$
$E(e_t e_s)=0 \qquad t \neq s$

a. Transform the model given above into a model in which all disturbance terms satisfy GM assumptions.
b. Find the best linear unbiased estimator of b_1 under the assumption that ρ is known.

Problem 8

You are interested in estimating a consumption function using annual data for the period 1930-1985.

$$C_t= b_0 + b_1 Y_1 + u_t$$

where

C=Consumption expenditures and
Y=Disposable income.

You are given the following OLS estimation results.

$$(1) \qquad C_t=\underset{(3.37)}{13.2} + \underset{(0.0127)}{0.88}Y_t \qquad R^2=0.988 \qquad DW=1.10$$

(2) $C_t = 10.8 + 0.78Y_t + 0.05Y_{t-1}$ $\quad R^2 = 0.990 \quad DW = 1.97$

$\qquad (3.01) \quad (0.01) \quad (0.115)$

(3) $C_t = 5.02 + 0.33Y_t + 0.639C_{t-1} - 0.118Y_{t-1}$ $\quad R^2 = 0.993 \quad DW = 2.15$

$\qquad (3.09) \quad (0.12) \quad (0.115) \quad (0.125)$

a. Test the presence of first order autocorrelation in (1) and (2).

b. What accounts for the serial correlation?