H. H. Kelejian and W. E. Oates, Introduction to Econometrics, Third Edition, Harper and Row Publishers, New York 1989.

1. Consider the model

$$C_t = b_0 + b_1 C_{t-1} + b_2 Y_t + \varepsilon_{1t}, \quad (1)$$

$$Y_t = I_t + C_t, \quad (2)$$

$$I_t = a_0 + a_1 Y_t + a_2 Y_{t-1} + a_3 r_t + \varepsilon_{2t}, \quad (3)$$

where C, I, Y, and r are, respectively, consumer expenditures, investment, income, and the interest rate. Assume that ε_1 and ε_2 are not autocorrelated and are independent of r_t.

a. List the endogenous variables and the predetermined variables in the model.

b. How would you estimate equation (1)?

c. How would you estimate equation (3)?

2. Take as a model of wage-price behavior:

$$\dot{W}_t = a_0 + a_1 (UN)_t + a_2 \dot{P}_t + \varepsilon_{1t},$$

$$\dot{P}_t = b_0 + b_1 \dot{M}_t + b_2 (UN)_t + b_3 \dot{W}_t + \varepsilon_{2t},$$

where

$\dot{W}$ = the percentage change in wages,
UN = the rate of unemployment,
$\dot{P}$ = the percentage change in prices,
$\dot{M}_t$ = the percentage change in the money supply, and
ε_1 and ε_2 = disturbance terms.

Assume that ε_{1t} and ε_{2t} have zero means, constant variances, are not autocorrelated, and are independent of $(UN)_t$ and $\dot{M}_t$.

a. Are the above equations identified? Explain.

b. Outline an estimation procedure for the identified equation.

3. Consider the model

$$L_t = a_0 + a_1 W_t + a_2 S_t + u_{1t}, \quad (1)$$

$$W_t = b_0 + b_1 L_t + b_2 P_t + u_{2t}, \quad (2)$$

where

L = the amount of labor employed,
W = the wage rate,
S = sales,
P = a measure of the productivity of labor.

a. Obtain the reduced-form equations for L_t and W_t.

b. Outline a technique for estimating equation (1).

4. Assume that the demand for shoes by an individual is described by

$$D_{it} = a_0 + a_1 P_t + a_2 D_{i(t-1)} + u_{it}, \tag{1}$$

where D_{it} is the ith individual's demand for shoes at time t, and P_t is the price he faces. Suppose that

$$u_{it} = \rho u_{i(t-1)} + \varepsilon_{it}, \qquad -1 < \rho < 1,$$

where ε_{it} has a zero mean, a constant variance, is not autocorrelated, and is independent of P_t and all of its lagged values.

a. Argue intuitively that the lagged dependent variable, $D_{i(t-1)}$ is correlated with the disturbance term.

b. Assume that equation (1) is not part of a system of equations. Demonstrate that it can nevertheless be estimated by TSLS.

5. Consider the following multiple-regression model:

$$Y_t = b_0 + b_1 X_{1t} + b_2 X_{2t} + u_{1t}, \tag{1}$$

$$X_{2t} = c_0 + c_1 Y_t + u_{2t}. \tag{2}$$

Show that, under our usual assumptions, $E(X_{2t}u_{1t}) \neq 0$.

6. Consider the wage-price model

$$\dot{W}_t = a_0 + a_1 \dot{P}_t + a_2(UN_t) + \varepsilon_{1t}, \tag{1}$$

$$\dot{P}_t = b_0 + b_1 \dot{W}_t + \varepsilon_{2t}, \tag{2}$$

where

$\dot{W}$ = the percentage change in money wages,
$\dot{P}$ = the percentage changes in prices, and
UN = the rate of unemployment.

a. Show that TSLS will not "work" if we attempt to estimate equation (1).

b. Does the TSLS procedure also break down if we attempt to estimate equation (2)? Explain.

7. Assume the following structural equation, which is part of a system of simultaneous equations:

$$Y_{1t} = b_0 + b_1 X_{1t} + b_2 Y_{2t} + b_3 Y_{3t} + u_{1t},$$

where Y_{1t}, Y_{2t}, and Y_{3t} are endogenous variables, and X_{1t} is a predetermined variable. Suppose that the complete system of which this equation is a member contains ten additional predetermined variables. However, suppose that we have observations on only one of them, say X_2.

a. Is the equation identified? Why or why not?

b. Can we estimate this equation by TSLS? Explain.

8. Consider the two-equation model

$$Y_{1t} = a_1 + b_1 X_t^2 + c_1 Y_{2t} + \varepsilon_{1t}, \tag{1}$$

$$Y_{2t} = a_2 + b_2 X_t + c_2 Y_{1t} + \varepsilon_{2t}, \tag{2}$$

where X_t is a predetermined variable and ε_1 and ε_2 satisfy our standard assumptions.

a. Are both equations identified? Why or why not?

b. Derive the reduced-form equations.
c. Outline a procedure for estimating the first equation in the above model.

9. Suppose that private investment spending is such that

$$I_{it} = a + b_1 r_{it} + b_2 S_{i(t-1)} + u_{it}, \qquad i = 1, \ldots, N,$$

$$r_{it} = r_t + b_3 I_{it} + \varepsilon_{it},$$

where

I_{it} = investment expenditures of the ith firm at time t,
r_{it} = the rate of interest it must pay for investment funds,
$S_{i(t-1)}$ = its sales in period $t - 1$, and
r_t = the economy-wide average interest rate for investment funds.

We assume that these N firms are large so that the level of their investment expenditure affects the interest rate they face. Assume the standard conditions concerning u_{it} and ε_{it}. Assume also that we have *only* cross-sectional data.

a. Discuss whether or not the equations are identified.
b. Obtain the reduced-form equation for I_{it}.