SOLUTIONS OF PROBLEM SET - V

(1) a) If $\gamma = \beta = 0$, and $a = 1$

$$Y_t = Y_{t-1} + e_t \Rightarrow \text{Pure Random walk}$$

$Y_1 = Y_0 + e_1$

$Y_2 = Y_0 + e_1 + e_2$

$\vdots$

$Y_t = Y_0 + \sum_{t=1}^{t} e_t$

$E(Y_t) = Y_0$

$var(Y_t) = \sum_{t=1}^{t} var(e_t) = t\sigma^2 \rightarrow$ not constant

So Y_t is ~~stationary~~ nonstationary

$\underbrace{Y_t - Y_{t-1}} = Y_{t-1} - Y_{t-1} + e_t$

$\Delta Y_t = e_t \Rightarrow$ So $\Delta Y_t \rightarrow$ is stationary

So Y_t is integrated of order 1.

b) If $\beta=0$ and $\alpha=1$

$Y_t = \gamma + Y_{t-1} + e_t \Rightarrow$ Random walk with drift

$Y_1 = \gamma + Y_0 + e_1$

$Y_2 = \gamma + \gamma + Y_0 + e_1 + e_2$

$Y_3 = \gamma + \gamma + \gamma + Y_0 + e_1 + e_2 + e_3$

$\vdots$

$Y_t = \gamma t + Y_0 + \Sigma e_t$

$E(Y_t) = \gamma t + Y_0 \Rightarrow$ not constat

$Var(Y_t) = t.\sigma^2 \Rightarrow$ not constat

so $Y_t \rightarrow$ nonstationary.

$\underbrace{Y_t - Y_{t-1}} = \gamma + \cancel{Y_{t-1}} - \cancel{Y_{t-1}} + e_t$

$\Delta Y_t = \gamma + e_t \Rightarrow$ so $\Delta Y_t \rightarrow$ stationary

c) If $\alpha=1$

$Y_t = \gamma + \beta t + Y_{t-1} + e_t \Rightarrow$ Random walk with drift and deterministic trend.

$Y_1 = \gamma + \beta.1 + Y_0 + e_1$

$Y_2 = \gamma + \beta.2 + \gamma + \beta.1 + Y_0 + e_1 + e_2$

$\vdots$

$Y_t = \gamma t + \beta.\left(\frac{t.(t+1)}{2}\right) + Y_0 + \Sigma e_t$

$E(Y_t) = \gamma t + \left[\frac{t \cdot (t+1)}{2}\right]\beta + Y_0 \Rightarrow$ not constant

$Var(Y_t) = t\sigma^2 \Rightarrow$ not constant

so $Y_t \Rightarrow$ nonstationary

$Y_t - Y_{t-1} = \gamma + \beta t + \cancel{Y_{t-1}} - \cancel{Y_{t-1}} + e_t$

$\Delta Y_t = \gamma + \beta t + e_t \Rightarrow$ so $\Delta Y_t \Rightarrow$ nonstationary

In order to solve the nonstationarity problem, we should also remove the deterministic trend.

2 a) For GDP $\Rightarrow$ use 1^{st} model

$$GDP_t = \beta GDP_{t-1} + \varepsilon_t$$

$$\Delta GDP_t = \underbrace{(\beta - 1)}_{\rho} GDP_{t-1} + \varepsilon_t$$

$H_0: \rho = 0 \Rightarrow \beta = 1$

$H_1: \rho \neq 0$ or $\rho < 0$

$Tau_{GDP} = 9.6427$

$\alpha = 0.05$, $T = 40$

Tau - table value for non constant term

$Tau = Z_{nc}$ for $\alpha = 0.05$ and $T = 50$ $= -1.95$

$|Tau| > |Z_{nc}| \Rightarrow$ Reject H_0

$\Rightarrow$ GDP is stationary

For RER $\Rightarrow$ use 2^{nd} model

$$RER_t = \gamma + \beta RER_{t-1} + \varepsilon_t$$

$H_0: \rho = 0$

$H_1: \rho \neq 0$

$Tau_{RER} = -1.92$

Tau statistic from table for $\alpha = 0.05$ and $T = 40$

$Z_c = -2.93$

$|Tau| < |Z_c| \Rightarrow$ DNR $H_0 \Rightarrow$ RER is nonstationary

For CPI, use 3rd model to ~~make~~ make unit root test.

$H_0: p = 1$

3rd model assumes that CPI has random walk with drift and deterministic trend.

So table value

$\alpha = 0.05$ & $T = 50 \Rightarrow \underbrace{\tau_{ct}}$ $= -3.50$

constant trend term

tau statistic of CPI $\Rightarrow \tau_{CPI} = -6.8493$

$|\tau| > |\tau_{ct}| \Rightarrow$ Reject $H_0 \Rightarrow$ CPI is stationary

b) CPI and GDP are integrated of order zero since they are stationary series However, we don't have enough information to determine the ~~integ~~ order of the integration of RER. We should also know whether ΔRER is stationary or not.

c) ~~All models have~~ There is no autocorrelation problem in both 3 models. However, we know that $p = (\beta - 1)$ should be equal to or lower than zero since β is between -1 and 1
$-1 \leq \beta \leq 1 \Rightarrow$ If you look the 1st model you will see that p is greater than zero which means that β is greater than 1. So

1st model or equation is not appropriate to conduct unit root test. 6

3 a) $H_0: \rho = 0$

$H_1: \rho \neq 0$

$\tau_{X_t} = -2.35$

$\alpha = 0.05$, $T' = 46 \Rightarrow \tau_c = -2.93$

$|\tau| = 2.35 < |\tau_c| = 2.93 \Rightarrow$ DNR H_0

$\Rightarrow$ Housing starts time series is nonstationary.

b) $X_t = \beta_0 + \beta_1 Y_{t-1} + u_t$

$\Delta X_t = \cancel{\beta} \beta_0 + \rho X_{t-1} + u_t$ ~~Y_t~~

$H_0: \rho = 0 \Rightarrow \beta = 1$

$H_1: \rho \neq 0$

$t = -2.35$

$t_{\alpha = 0.05/2,\ T' = (47-1)} \cong 2.00$

$|t| = 2.35 > t_{tab} \Rightarrow$ Reject $H_0 \Rightarrow X_t$ series is stationary. However, under the null hypothesis that $\rho = 0$ (i.e $\beta = 1$), the t value of the estimated coefficient of X_{t-1} does not follow the t distribution even in large samples. So, we can't use t-test to conduct unit root test. The result of using t-test will be misleading.

c) $H_0: \rho = 0$

z of the coefficiet of $\Delta X_{t-1} = -5.89$

z_c for $\alpha = 0.05$ and $T' = (47-2) = -3.50$

$T' = 45 \sim 50$

↳ table value

z_c

$|z| = 5.89 > |z_c| = 3.50 \Rightarrow$ Reject $H_0 \Rightarrow \Delta X_t$ is stationary.

So X_t is integrated of order 1

ΔX_t is " " " 0.

So taking first difference of X_t makes series stationary.

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a) 1st equation has autocorrelation problem. $DW = 1.03$

$\alpha = 0.05$
$T' = (50-1) = 49$
$k' = 1$

$d_L = 1.503$
$d_u = 1.585$

$0 < DW = 1.03 < d_L$

$\Rightarrow$ so $\exists$ AC problem in the first equation we can't use this equation to conduct unit root test for y_t series. Also, 4th equation has AC problem.

we know that a (i.e. $\beta-1$) should be equal to zero or lower than zero.

$$Z = \frac{\hat{a}-0}{se(\hat{a})}$$ ⇒ so the tau value should be smaller than zero in other word it should be a negative value. However, the tau value of the coefficient of y_{t-1} in the 2nd equation

$Z = 1.20$

which is greater than zero. Therefore we should not use 2nd equation.

use Eq (3) ⇒ $\Delta y_t = \alpha y_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + u_t$

$LM(AR(1)) = \cancel{1.04} < \chi^2_{\alpha=0.05,1} = 3.84 \Rightarrow \nexists$ AC problem.

$H_0: \alpha = 0$

$H_1: \alpha \neq 0$

$Z = -0.7$ $\quad Z_{nc}\ (\alpha = 0.05) = -1.95$

$|Z| = 0.7 < |Z_{nc}| = 1.95 \Rightarrow$ Do not reject H_0

$\Rightarrow y_t$ is nonstationary

use Eq (5) and Eq (6)

$\Rightarrow \Delta^2 y_t = \alpha \Delta y_{t-1} + \alpha_1 \Delta^2 y_{t-1} + \varepsilon_t \quad \Rightarrow$ Eq (5)

$H_0: \alpha = 0$

$H_1: \alpha \neq 0$

$Z = -2.7$ $\quad Z_{nc}\ (\alpha = 0.05) = -1.95$

$|Z| = 2.7 > |Z_{nc}| = 1.95 \Rightarrow RH_0 \Rightarrow$

Δy_t is stationary.

$$Eq(6) \rightarrow \Delta^2 y_t = \alpha \Delta y_{t-1} + \alpha_1 \Delta^2 y_{t-1} + \alpha_2 \Delta^2 y_{t-2} + \varepsilon_t$$

$H_0: \alpha = 0$
$H_1: \alpha \neq 0$

$Z = -2.3$ $\quad$ $Z_{nc}(\alpha = 0.05) = -1.95$

$|Z| = 2.3 > |Z_{nc}| = 1.95 \Rightarrow R\,H_0 \Rightarrow \Delta y_t$ is stationary.

$\Rightarrow \Delta y_t \longrightarrow I(0)$
$\quad y_t \longrightarrow I(1)$

b) $p \rightarrow$ lag length of augmentation.

$$p_{max} = \text{integer}\left[12 \cdot \left(\frac{T}{100}\right)^{1/4}\right]$$

to find lag length, we should ~~follow~~ do the following steps

(1) Set an upper bound p_{max} for p

(2) Estimate ADF test regression with $p = p_{max}$

(3) If the absolute of the t-statistic for testing the significance of the last lagged difference is greater than 1.6 then set $p = p_{max}$ and carry out the ADF unit root test of $H_0: \alpha = 0$ vs $H_A: \alpha \neq 0, \alpha < 0$. otherwise (if not significant), reduce the lag length by one and repeat the process.

However, the p_{max} should be 10 according to Schwert. but we do not have a model includes $p_{max} = 10$. So we can't apply the ~~above~~ given procedure to find appropriate lag length. But

~~If you look the (…)~~

We use Eq(3) to test unit root which is appropriate so we can say 2 lenght of augmentation is appropriate to test.

(5)

a) 1^{st} model is estimated to find the relation between $\ln M1t$ and $\ln GDPt$. The coefficient of $\ln GDPt$ shows the elasticity. It is individually significant.

$DW = 0.3254$ closes to zero which tells us that ∃ AC problem in the 1^{st} model.

The 2^{nd} model is estimated by ~~taking~~ ~~the first~~ regressing the first difference of $\ln M1t$ on the first diffence of $\ln GDPt$. The estimated slope coefficient is still significant but. it drops sharply. The DW value of the second model suggests that ∄ AC problem in the 2^{nd} model.

b) According to Granger and Newbold, $R^2 > DW$ is a good rule of thum to suspect that the estimated regression spurious.

so in the 1^{st} equation, $R^2 > DW$ ⇒ so we suspect that regression 1 is spurious.

c) in the 2^{nd} regression, $R^2 < DW$

⇒ we donot suspect that 2^{nd} regression is spurious.

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d) 3^{rd} equation is estimated to find whether $\ln M1_t$ and $\ln GDP_t$ are cointegrated or not.

$$\Delta\hat{u}_t = -0.1958\, \hat{u}_{t-1}$$
$$(-2.2521)$$

$$u_t = \beta u_{t-1} + v_t$$

$$\Delta u_t = \rho u_{t-1} + v_t$$

$$\rho = (\beta - 1)$$

$$H_0: \rho = 0$$

$z = -2.2521$

z_{nc} for $\alpha = 0.05$

$T' = (52-1) = 51$

$z_{nc} = -1.95$

$|z| = 2.2521 > |z_{nc}| = 1.95$

$\Rightarrow$ Reject H_0 $\Rightarrow$ u_t series ~~is~~ stationary.

So $\ln M1_t$ and $\ln GDP_t$ are cointegrated. In other words, there is long-run relationship between $\ln M1_t$ and $\ln GDP_t$. Hence, we should change our conclusion in b and we should conclude that 1^{st} model is not spurious.

e) This regression is an Error Correction model. we find that there is log-term relationship between $\ln M1_t$ and $\ln GDP_t$. This regression is estmated to find whether this logg-run relationship between two variables holds in the shortrun or not

If this long-run relationship does not hold in the short-run, how adjustment made in the short-run to achieve long-run relation.

(13)

$$\Delta \ln M1_t = \alpha_0 + \alpha_1 \Delta \ln GDP_{t-1} + \alpha_2 \Delta \ln M1_{t-1} + \lambda u_{t-1} + v_t$$

we will test whether the coefficient of u_{t-1} is significant or not.

$H_0: \lambda = 0$

~~te~~ $H_1: \lambda \neq 0$

$t = -3.9835 \Rightarrow$ use t-distribution table to conduct test.

$\alpha = 0.05$

$t_{0.025,\ T' = (52-1=51)} \cong 2.00$

$|t| = 3.9835 > t_{table} = 2.00 \Rightarrow$ Reject Ho

$\Rightarrow$ It is statistically significant

$\Rightarrow$ this long-run relationship does not hold in the short-run.

$\Rightarrow$ The coefficient of u_{t-1} shows how adjustment made in the short-run to achieve long-run relation. It tell us that the 7.11 percent of the error has been made in previous period is eliminated in this period.

~~In order to~~

$LM(AR(1)) = 1.9817 \Rightarrow v_t = \rho v_{t-1} + \varepsilon_t$

$H_0: \rho = 0$

$H_1: \rho \neq 0$

$\chi^2_{\alpha=0.05, 1} = 3.84$

$LM(AR(1)) = 1.9817 < \chi^2_{\text{table-value}} = 3.84$

$\Rightarrow$ Do not reject $H_0 \Rightarrow \nexists$ AC problem in this model

so $\quad Cov(v_t, v_s) = 0$
$\quad t \neq s$

White $= 1.44 \Rightarrow$ conduct heteroscedasticity test.

$\chi^2_{\alpha=0.05, 9} = 16.9190$

White $< \chi^2 = 16.9190 \Rightarrow$ Do not reject H_0

$\Rightarrow \nexists$ heteroscedasticity problem

$Var(v_t) = \sigma^2$

H_0: v_t is normally distributed

H_1: v_t is not normally distributed

$\chi^2_{\alpha=0.05, 2} = 5.99$

$JB = 0.847$

$JB < \chi^2_{(2)} = 5.99 \Rightarrow$ Do not Reject H_0.

$\Rightarrow E(v_t) = 0$

so error term of this regression is white-noise.

6 a) $ADF(m_t) = -1.85$

$$\Delta m_t = \alpha_0 + \theta m_{t-1} + \sum_{j=2}^{q} a_j \Delta m_{t-j+1} + e_t$$

Z_c for $\alpha = 0.05$ ~~γ~~ $T' = 24 = -3.00$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$|ADF(m_t)| = 1.85 < |Z_c| = 3.00$

$\Rightarrow$ Do not reject $H_0 \Rightarrow m_t$ is nonstationary

$ADF(\Delta m_t) = -7.17$

$$\Delta^2 m_t = \alpha_0 + \theta \Delta m_{t-1} + \sum_{j=2}^{q} \delta_j \Delta^2 m_{t-j+1} + \varepsilon_t$$

$Z_c = -3.00$

$H_0: \theta = 0$
$H_1: \theta \neq 0$

$|ADF(\Delta m_t)| = 7.17 > |Z_c| = 3.00 \Rightarrow$ Reject H_0

$\Rightarrow \Delta m_t$ is stationary

so $\Delta m_t \longrightarrow I(0)$

$m_t \rightarrow I(1)$

y_t series

$H_0: \theta = 0$
$H_1: \theta \neq 0$

$ADF(y_t) = -2.15$ $\quad Z_c = -3.00$

$|ADF(y_t)| = 2.15 < |Z_c| = 3.00 \Rightarrow$ DN Reject H_0

$\Rightarrow y_t$ is nonstationary

$ADF(\Delta y_t) = -6.38$ $H_0: \theta = 0$
$H_1: \theta \neq 0$

$z_c = -3.00$

$|ADF(\Delta y_t)| = 6.38 > |z_c| = 3.00$

$\Rightarrow$ Reject $H_0 \rightarrow \Delta y_t$ is stationary

, so $\Delta y_t \longrightarrow I(0)$
$y_t \longrightarrow I(1)$

Hence, y_t and m_t are integrated of the same order.

b) To find whether m_t and y_t are cointegrated $\Rightarrow$ check for stationarity of u_t

$$\Delta u_t = \alpha_0 + \theta u_{t-1} + \sum_{j=2}^{q} a_i \Delta u_{t-j+1} + e_t$$

$H_0: \theta = 0$
$H_1: \theta \neq 0$

$ADF(\hat{u}_t) = -1.75$ $z_c = -3.00$

$|ADF(\hat{u}_t)| = 1.75 < |z_c| = 3.00 \Rightarrow$ Do not reject H_0

$\Rightarrow u_t$ is nonstationary

$\Rightarrow m_t$ and y_t are not cointegrated

c) Since $\nexists$ cointegration, we can not estimate with error correction mechanism.

$$\Delta m_t = \alpha_0 + \alpha_1 \Delta y_t + \lambda (m_{t-1} - \delta y_{t-1})$$

Since $\nexists$ cointegration, ECM for this model is invalid but we still can estimate first differences

$$\Delta m_t = \alpha_0 + \beta \Delta y_t + \varepsilon_t$$

7 a) AC series

use (7) and (8) ~~AAC~~

Eq. (7) $\Rightarrow Z = -3.01$

$\Delta AC_t = \alpha_0 + \alpha_1 t + \theta AC_{t-1} + \beta \Delta AC_{t-1} + \varepsilon_t$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

Z_{ct} (for $\alpha = 0.05$ and $T' = 44 - 2 = 42$) $= -3.50$

$|Z| = 3.01 < |Z_{ct}| = 3.50 \Rightarrow$ DNR H_0

$\Rightarrow AC_t$ is nonstationary

Eq.(8) $\Rightarrow Z = -5.24$

$Z_c (\alpha = 0.05, T' = 41) = -3.50$

$|Z| = 5.24 > |Z_c| = 3.50 \Rightarrow$ Reject $H_0 \Rightarrow$

$\Rightarrow \Delta AC_t$ is stationary

$\Rightarrow \Delta AC_t \rightarrow I(0)$

$AC_t \rightarrow I(1)$

AS series

use (9), (10) and (11)

Equation (10) is an equation estimated under DF. So we should check whether there is Autocorrelation problem or not

$LM(AR(1)) = 11.8$ $\qquad u_t = \rho u_{t-1} + e_t$

$H_0: \rho = 0$
$H_1: \rho \neq 0$

$\chi^2_{\alpha=0.05,1} = 3.84$

$\Rightarrow LM(AR(1)) = 11.8 > \chi^2_2 = 3.84 \Rightarrow$ Reject $H_0 \Rightarrow \exists$ AC in Eq. (10)

$\Rightarrow$ So, we can't use Eq. (10) to conduct unit root tes. Isteand of Eq(10), we use Eq. (9) which has not AC problem.

$LM(AR(1)) = 1.31 < \chi^2_2 = 3.84 \Rightarrow \nexists$ AC problem in Eq. (9)

Eq. (9) $\quad \tau = -2.90$ $\qquad H_0: \theta = 0$ $\qquad \theta \rightarrow$ is coefficient of AS_{t-1} in Eq(9)
$H_1: \theta \neq 0$

$\tau_{ct}(\alpha = 0.05, T' = 44 - 3 = 41) = -3.50$

$|\tau| = 2.90 < |\tau_{ct}| = 3.50 \Rightarrow$ DNR H_0

$\Rightarrow$ AS is nonstationary.

Eq (11) $\rightarrow$ $H_0: \theta = 0$ $\quad$ $\theta \Rightarrow$ coefficient of ΔAS_{t-1}

$H_1: \theta \neq 0$ $\quad$ in Eq (11)

$Z = -8.88$

$Z_c(\alpha = 0.05, T' = 41) = -2.93$

$|Z| = 8.88 > |Z_c| = 2.93 \Rightarrow$ Reject H_0

$\Rightarrow \Delta AS_t$ is stationary.

$\Delta AS_t \rightarrow I(0)$

$AS_t \rightarrow I(1)$

OE series

use Eq. (5) and (6)

Eq. (5) $\Rightarrow \Delta OE_t = \alpha_0 + \theta OE_{t-1} + e_t$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z = -1.83$ $\quad$ $Z_c = -2.93$

$(\alpha = 0.05$

$T' = 43)$

$|Z| = 1.83 < |Z_c| = 2.93 \Rightarrow$ DNR $H_0 \Rightarrow OE_t$ is nonstationary

Eq (6) $\Rightarrow \Delta^2 OE_t = \alpha_0 + \theta \Delta OE_{t-1} + \varepsilon_t$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z = -8.14$ $\quad$ $Z_c(\alpha = 0.05, T' = 42) = -2.93$

$|z| = 8.14 > |z_c| = 2.93 \Rightarrow$ Reject H_0

$\Rightarrow \Delta OE_t$ is stationary

$\Rightarrow \Delta OE_t \rightarrow I(0)$

$OE_t \longrightarrow I(1)$

b) To conduct ~~test~~ of cointegration, use Eq. (2)

$\Delta u_t = \theta u_{t-1} + \varepsilon_t$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

Since all series in the first equation has the integrated of same order, we can conduct cointegration test

$z = -5.99$

$z_c\ (\alpha = 0.05, T' = 43) = -2.93$

$|z| = 5.99 > |z_c| = 2.93 \Rightarrow$ Reject H_0

$\Rightarrow u_t \rightarrow$ stationary.

So these series are cointegrated. $\exists$
(AS, OE, AC)

long-run relationship between those variables.

c) Eq. (13) is an ECM model. This model is estimated to investigate whether the long-run relationship between these variables hold in the short-run or not.

$$\Delta AC_t = \alpha_0 + \alpha_1 \Delta AC_{t-1} + \alpha_2 \Delta AS_{t-1} + \alpha_3 \Delta OE_{t-1} + \lambda u_{t-1} + v_t$$

$H_0: \lambda = 0$
$H_1: \lambda \neq 0$

use t-statistic (student's t-distribution)

$t = -6.45$

$\alpha = 0.05$

$t_{0.025, 43} \cong 2.00$

$|t| = 6.45 > t_{table} = 2.00$

$\Rightarrow$ Reject H_0

$\Rightarrow \lambda$ is ~~significantly~~ statistically significant.

$\lambda \Rightarrow$ 94 percent of the error has been made previous period is eliminated in this period.

d)

e) Eq(3) $\Rightarrow \Delta AC_t = \alpha_0 + \alpha_1 \Delta AC_{t-1} + \alpha_2 \Delta AS_{t-1} + \alpha_3 \Delta OE_{t-1}$

$+ \lambda u_{t-1} + \varepsilon_t$

H_0: ε_t is normally distributed

H_1: ε_t is not " ".

$JB = 0.7761$ $\qquad \chi^2_{\alpha=0.05,2} = 5.99$

$JB < \chi^2_2 \Rightarrow$ DNR $H_0 \Rightarrow \varepsilon_t$ is normally distributed.

$\underline{\underline{E(\varepsilon_t) = 0}}$

white = 1.61 $\Rightarrow$ heteroscedastricity test.

$\chi^2_{\alpha=0.05,14} = 23.68$

white = 1.61 $< \chi^2_{14} \Rightarrow$ DNR $H_0 \Rightarrow \nexists$ heteroscedastricity

$Var(\varepsilon_t) = E(\varepsilon_t^2) = \sigma^2$

LM = 0.97 for AR(1) $\qquad H_0: \rho = 0 \qquad \varepsilon_t = \rho\varepsilon_{t-1} + v_t$

$H_1: \rho \neq 0$

$\chi^2_{\alpha=0.05,1} = 3.84$

$LM(AR(1)) = 0.97 < \chi^2_1 \Rightarrow$ DNR $H_0 \Rightarrow \nexists$ AC

$Cov(\varepsilon_t, \varepsilon_s) = 0$

$t \neq s$

$\Rightarrow$ so ε_t is white noise.

10 y_t series

$$\Delta y_t = \theta y_{t-1} + \sum_{j=2}^{q} \beta_i \Delta y_{t-j+1} + \varepsilon_t$$

$H_0: \theta = 0$ $\alpha = 0.05$

$H_1: \theta \neq 0$

$ADF(y_t) = 1.21$ $Z_{nc} = -1.95$

$|ADF(y_t)| = 1.21 < |Z_{nc}| = 1.95 \Rightarrow$ Do not reject H_0

$\Rightarrow y_t$ is nonstationary

$$\Delta^2 y_t = \theta \Delta y_{t-1} + \sum_{j=2}^{q} \beta_j \Delta^2 y_{t-j+1} + \varepsilon_t$$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z_{nc}\ (\alpha = 0.05) = -1.95$

$|ADF(\Delta y_t)| = 4.05 > |Z_{nc}| = 1.95 \Rightarrow$ Reject H_0

$\Rightarrow \Delta y_t$ is stationary

$\Rightarrow \Delta y_t \rightarrow I(0)$

$y_t \rightarrow I(1)$

X_{1t} series

$H_0: \theta = 0$

$H_1: \theta = 0$

$|ADF(x_{1t})| = 5.24 > |Z_{nc}(\alpha = 0.05)| = 1.95$

$\Rightarrow$ Reject H_0

$\Rightarrow x_{1t}$ is stationary

$\Rightarrow x_1 \rightarrow I(0)$

x_{2t} series

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z_{nc} (\alpha = 0.05) = -1.95$

$|ADF(x_{2t})| = 0.11 < |Z_{nc}| = 1.95 \Rightarrow$ DNR $H_0 \Rightarrow x_{2t}$ is nonstationary

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z_{nc} (\alpha = 0.05) = -1.95$

$|ADF(\Delta x_{2t})| = 7.27 > |Z_{nc}| = 1.95 \Rightarrow$ Reject H_0

$\Rightarrow \Delta x_{2t}$ is stationary

$\Rightarrow \Delta x_{2t} \rightarrow I(0)$

$x_{2t} \rightarrow I(1)$

in model (a), $y_t \rightarrow I(1)$, $x_{1t} \rightarrow I(0)$, $x_{2t} \rightarrow I(1)$ } since they are not integrated of same order, it does not make sense to test for cointegration

in model (b), $y_t \rightarrow I(1)$, $x_{1t} \rightarrow I(0)$ } since they are not integrated of same order, it does not make sense to test for cointegration

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a) p_t series

$$\Delta p_t = \theta p_{t-1} + \sum_{j=1}^{q} \beta_j \Delta p_{t-j} + \varepsilon_t$$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z_{nc}(\alpha = 0.01) = -2.62$

$ADF(p_t) = -0.25$ $\quad Z_{nc}(\alpha = 0.05, T=30) = -1.95$

$|ADF(p_t)| = 0.25 < |Z_{nc}| = 1.95 \Rightarrow$ DNR H_0

$< |Z_{nc}| = 2.62 \Rightarrow p_t$ is nonstationary

$$\Delta^2 p_t = \theta \Delta p_{t-1} + \sum_{j=1}^{q} \beta_j \Delta^2 p_{t-1} + \varepsilon_t$$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$ADF(\Delta p_t) = -8.11$ $\quad Z_{nc}(\alpha = 0.05) = -1.95$

$Z_{nc}(\alpha = 0.01) = -2.62$

$|ADF(\Delta p_t)| = 8.11 > |Z_{nc}| = 1.95 \Rightarrow$ Reject H_0

$= 8.11 > |Z_{nc}| = 2.62 \Rightarrow \Delta p_t$ is stationary

$\Rightarrow \Delta p_t \longrightarrow I(0)$

$p_t \longrightarrow I(1)$

w_t series

$$\Delta w_t = \theta w_{t-1} + \sum_{j=1}^{q} \beta_j \Delta w_{t-j} + \varepsilon_t$$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$ADF(w_t) = 2.18$ $\quad Z_{nc}(\alpha = 0.05) = -1.95$

$Z_{nc}(\alpha = 0.01) = -2.62$

$|ADF(w_t)| = 2.18 < |Z_{nc}(\alpha=0.01)| = 2.62 \Rightarrow$ DNR H_0

$\Rightarrow w_t$ is nonstationary

$$\Delta^2 w_t = \theta \Delta w_{t-1} + \sum_{j=1}^{9} \beta_j \Delta^2 w_{t-j} + \varepsilon_t$$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$Z_{nc}(\alpha = 0.05) = -1.95$

$Z_{nc}(\alpha = 0.01) = -2.60$

$ADF(\Delta w_t) = -3.37$

$|ADF(\Delta w_t)| = 3.37 > |Z_{nc}| = 1.95$

$> |Z_{nc}| = 2.60$

$\Rightarrow$ Reject Ho

$\Rightarrow \Delta w_t$ is stationary

$\Rightarrow \Delta w_t \longrightarrow I(0)$

$w_t \longrightarrow I(1)$

b) since p_t and w_t are integrated of same order, necess condition for cointegration of p_t and w_t is met.

c) in non-homogenous model $\Rightarrow R^2 = 0.85 > DW = 0.57$

$\Rightarrow$ so we suspect that non-homogenous model is spurious. check cointegration !!

d) $ADF(\hat{v}_t) = -0.58$

$Z_{nc}(\alpha = 0.05) = -1.95$

$Z_{nc}(\alpha = 0.01) = -2.60$

$|ADF(\hat{v}_t)| = 0.58 < |Z_{nc}| = 1.95$

$< |Z_{nc}| = 2.60$

$\Delta v_t = \theta v_{t-1} + \varepsilon_t$

$H_0: \theta = 0$

$H_1: \theta \neq 0$

$\Rightarrow$ Do not reject Ho

$\Rightarrow v_t$ is nonstationary $\Rightarrow$ ADF test does not confirm

... hypothesis in the

non-homogenous model. In other words, there is no cointegration in the non-homogenous model.

e) under the homogeneity, we should check cointegration in the first equation. (homogenous model)

$$\Delta u_t = \theta u_{t-1} + \sum_{j=1}^{J} \beta_j \Delta u_{t-j} + \varepsilon_t$$

$H_0 : \theta = 0$

$H_1 : \theta \neq 0$

$ADF(\hat{u}_t) = -4.97$ $\quad$ $Z_{nc}(\alpha = 0.05) = -1.95$

$Z_{nc}(\alpha = 0.01) = -2.60$

$$\left.\begin{array}{l} |ADF(\hat{u}_t)| = 4.97 > |Z_{nc}| = 1.95 \\ \phantom{|ADF(\hat{u}_t)| = 4.97} > |Z_{nc}| = 2.60 \end{array}\right\} \Rightarrow \text{Reject } H_0 \Rightarrow u_t \text{ is stationary.}$$

So p_t and w_t are cointegrated under homogeneity.

f) we should prefer to use the homogeneity assumption to estimate the wage-adjustment equation since p_t and w_t are cointegrated under homogeneity but not cointegrated in the non-homogenous model.