VECTOR AUTOREGRESSIVE (VAR) MODELS

It is quite common in economics to have models in which some variables are not only explanatory variables for a given dependent variables, but are also explained by the variables that they are used to determine.

In these cases, we have models of simultaneous equations, in which it is necessary to identify endogeneous and exogeneous variables.

The decision regarding such a differentiation among variables was heavily criticized by Sims (1980). According to Sims, if there is simultaneity among a number of variables, then all these variables should be treated in the same way.

In other words, there should be no distinction between endogeneous and exogeneous variables.

Therefore, once this distinction is abandoned, all variables are treated as endogeneous. This means that in its general reduced form each equation has the set of regressors, which leads to the development of VAR models.

The VAR MODEL

When we are not confident that a variable really is exogeneous, each variable has to be treated symmetrically.

Take, for example, the time series Y_t that is affected by current and past values of X_t and; simultaneously, the time series X_t to be a series that is affected by current and past values of the Y_t series

(1) $Y_t = \beta_{10} - \beta_{12} X_t + \gamma_{11} Y_{t-1} + \gamma_{12} X_{t-1} + u_{yt}$

(2) $X_t = \beta_{20} - \beta_{21} Y_t + \gamma_{21} Y_{t-1} + \gamma_{22} X_{t-1} + u_{xt}$

where we assume that both Y_t and X_t are stationary [we will see this issue later] and u_{yt} and u_{xt} are uncorrelated white-noise error terms.] u_{xt} and u_{yt} → are also called structural shocks.

Equations (1) and (2) constitute a first-order VAR model, because the longest lag is unity.

These equations are not reduced-form equations, since Y_t has a contemporaneous impact on X_t (given by $-\beta_{21}$) and X_t has a contemporaneous impact on Y_t (given by $-\beta_{12}$).

Rewriting the system using matrix algebra, we get

$$\begin{cases} Y_t + \beta_{12} X_t = \beta_{10} + \gamma_{11} Y_{t-1} + \gamma_{12} X_{t-1} + u_{yt} \\ \beta_{21} Y_t + X_t = \beta_{20} + \gamma_{21} Y_{t-1} + \gamma_{22} X_{t-1} + u_{xt} \end{cases}$$

→ Structural VAR (SVAR) or the primitive system

$$(3)\quad \underbrace{\begin{bmatrix} 1 & \beta_{12} \\ \beta_{21} & 1 \end{bmatrix}}_{\underline{B}} \underbrace{\begin{bmatrix} y_t \\ x_t \end{bmatrix}}_{\underline{Z}_t} = \underbrace{\begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix}}_{\underline{\Theta}_0} + \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}}_{\underline{\Theta}_1} \underbrace{\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix}}_{\underline{Z}_{t-1}} + \underbrace{\begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}}_{\underline{u}_t}$$

$$(4)\quad \underline{B}\,\underline{Z}_t = \underline{\Theta}_0 + \underline{\Theta}_1 \underline{Z}_{t-1} + \underline{u}_t$$

} Structural VAR (SVAR) or the Primitive System

Multiplying both sides by $\underline{B}^{-1}$ yields

$$\underline{Z}_t = \underbrace{\underline{B}^{-1}\underline{\Theta}_0}_{} + \underbrace{\underline{B}^{-1}\underline{\Theta}_1}_{} \underline{Z}_{t-1} + \underbrace{\underline{B}^{-1}\underline{u}_t}_{}$$

(5) (VAR in standard form unstructured VAR = UVAR)

$$\boxed{\underline{Z}_t = \underline{A}_0 + \underline{A}_1 \cdot \underline{Z}_{t-1} + \underline{e}_t}$$

Reduced form

For purposes of notational simplification we can denote as a_{i0} the i^{th} element of the vector $\underline{A}_0$; a_{ij} the element in row i and column j of the matrix $\underline{A}_1$; and e_{it} as the i^{th} element of the vector $\underline{e}_t$

<u>Not</u>: To decide the # of lags we can use Akaike or Schwarz criterions and choose with lowest ones.

Using this notation, we can rewrite the VAR model as;

impulses or innovations or shocks

$$(6) \quad y_t = a_{10} + a_{11} y_{t-1} + a_{12} x_{t-1} + e_{1t}$$

$$(7) \quad x_t = a_{20} + a_{21} y_{t-1} + a_{22} x_{t-1} + e_{2t}$$

Here the new error terms; e_{1t} and e_{2t}, are composites of the two shocks u_{yt} and u_{xt}. e_{1t} and e_{2t} are also white noise processes.

Pros and Cons of the VAR Models

The VAR model approach has some very good chracteristics. First, it is very simple.

(1) The econometrician does not have to worry about which variables are endogeneous or exogeneous.

(2) Estimation is also very simple, in the sense that each equation can be <u>estimated seperately</u> with the usual <u>OLS</u> method.

→

(3) Forecasts obtained from VAR models are in most cases better than those obtained from for more complex simultaneous equation models

However, VAR models have also faced severe criticism over various points

(1) Firstly, they are <u>atheoretic</u>, in that they are not based on any economic theory. In addition, "everything causes everything."

(2) Another problem is the loss of degrees of freedom. If we suppose that we have a 3 variable VAR model and decide to include 12 lags from each variable in each equation, the degrees of freedom will decline by 36 plus 1 (for intercept), i.e., by 37. If the sample size is not sufficiently large, estimating will be very problematic.

(3) The obtained coefficients of the VAR model are difficult to interpret because of their lack of any theoretical background.

To overcome this critisism, the advocates of VAR models estimate so-called impulse response functions. The impulse response function examines the response of the dependent variable in the VAR to shocks in the error terms. The difficult issue here, however, is defining the shocks.

The general view is that we would like to shock the structural errors (u_{yt} and u_{xt}) which we can interpret easily as a shock to a particular part of a structural model. However, we only observe the reduced-form errors in Equations (6) and (7), and these are made up of a combination of structural errors

$$e_{1t} = \frac{u_{yt} + \beta_{12} u_{xt}}{1 - \beta_{12}\beta_{21}}$$

$$e_{2t} = \frac{u_{xt} + \beta_{21} u_{yt}}{1 - \beta_{12}\beta_{21}}$$

So we have to disentangle the structural errors in some way, and this is known as the identification problem. There are a variety of ways of doing this, however the different methods can give rise to quite different results and there is no objective statistical criteria for choosing between these different methods.

Causality Tests

Causality in econometrics is somewhat different from the concept in everyday use; it refers more to the ability of one variable to predict (and therefore cause) the other. The relationship between these variables can be captured by a VAR model. In this case it is possible to state that

(1) y_t causes x_t

(2) x_t causes y_t

(3) There is a bi-directional feedback

(4) Two variables are independent

Granger (1969) developed a relatively simple test that defined causality as follows:

"a variable Y_t is said to be Granger cause X_t if X_t can be predicted with greater accuracy by using past values of the Y_t variable rather than not using such past values, all other terms remaining unchanged"

The Granger Causality Test

The Granger causality test for the case of two stationary variables Y_t and X_t involves as a first step the estimation of the following VAR model

$$Y_t = a_1 + \sum_{i=1}^{n} \overset{\text{sig.}}{\beta_i} X_{t-i} + \sum_{j=1}^{m} \overset{\text{not sig.} \rightarrow X \text{ granger causes } Y}{\gamma_j} Y_{t-j} + e_{1t} \quad (1)$$

$$X_t = a_2 + \sum_{i=1}^{n} \underset{\text{not sig}}{\theta_i} X_{t-i} + \sum_{j=1}^{m} \underset{\text{signif.} \rightarrow Y \text{ Granger cause } X}{\delta_j} Y_{t-j} + e_{2t} \quad (2)$$

where it is assumed that both u_{yt} and u_{xt} are uncorrelated white-noise error terms.

In this model we can have the following different cases:

Case 1 The lagged X_t terms in equation (1) may be statistically different from zero as a group, and the lagged Y_t terms in equation (2) not statistically different from zero. In this case we see X_t (Granger) causes Y_t.

Case 2 The lagged Y_t terms in Eq (2) may be statistically different from zero as a group, and the lagged X_t terms in Eq (1) not statistically different from zero. In this case we see that Y_t (Granger) causes X_t.

Case 3 Both sets of X_t and Y_t terms are statistically different from zero in Eq (1) and (2), so that there is bi-directional causality

Case 4 Both sets of X_t and Y_t terms are not statistically different from zero in Eq (1) and (2), so that X_t is independent of Y_t.

The Granger causality test, then involves the following procedure. First, estimate the VAR model given by the Equations (1) and (2). Then check the significance of the coefficients and apply variable deletion tests, first in the lagged X terms for Equation (1) and then in the lagged Y terms for Eq. (2).

Steps are as follows

Step 1 Regress Y_t on lagged Y terms as follows:

$$Y_t = a_1 + \sum_{j=1}^{m} \gamma_j \cdot Y_{t-j} + e_{1t} \qquad (3)$$

and obtain the SSR of this regression (restricted model) and label it as SSR_R.

Step 2 Regress Y_t on lagged Y terms plus lagged X terms as in Equation (1)

$$Y_t = a_1 + \sum_{i=1}^{n} \beta_i X_{t-i} + \sum_{j=1}^{m} \gamma_j Y_{t-j} + e_{1t} \qquad (1)$$

and obtain the SSR of this regression (unrestricted model) and label it as SSR_U.

Step 3 Set the H_0 and H_A as:

H_0: $\sum_{i=1}^{\hat{}} \beta_i = 0$ (or X_t does not cause Y_t)

H_A: $\sum_{i=1}^{\hat{}} \beta_i \neq 0$ (or X_t does cause Y_t)

Step 4 calculate the Q statistic

$$Q = \frac{(SSR_R - SSR_U)/p}{SSR_U/(T-k-1)}$$

where $k = m+n$ and p is the # of restrictions (so $p = n$)

$$Q \sim F^{\alpha}_{p,\, T-k-1}$$

Step 5 If $Q > F^{\alpha}_{p,T-k-1}$ $\Rightarrow$ R H_0 and conclude that X Granger causes Y

GRANGER CAUSALITY

- We have to be aware of that Granger causality does not equal to what we usually mean by causality. For instance, even if x_1 does not cause x_2, it may still help to predict x_2, and thus Granger-causes x_2 if changes in x_1 precedes that of x_2 for some reason.
- A naive example is that we observe that a dragonfly flies much lower before a rain storm, due to the lower air pressure. We know that dragonflies do not cause a rain storm, but it does help to predict a rain storm, thus Granger-causes a rain storm.