supply of the various inputs. →The demand for factors by firms depends not only on the state of technology but also on the demands for the final goods they produce. →The demands for these goods depend on consumers' incomes, which, as we saw, depend on the demand for the factors of production. This circular interdependence of the activity within an economic system can be illustrated with a simple economy composed of two sectors, a consumer sector, which includes households and a business sector, which includes firms.[1] It is assumed that: (*a*) all production takes place in the business sector; (*b*) all factors of production are owned by the households;[2] (*c*) all factors are fully employed; (*d*) all incomes are spent.

The economic activity in the system takes the form of two flows between the consumer sector and the business sector: a real flow and a monetary flow (figure 22.1).

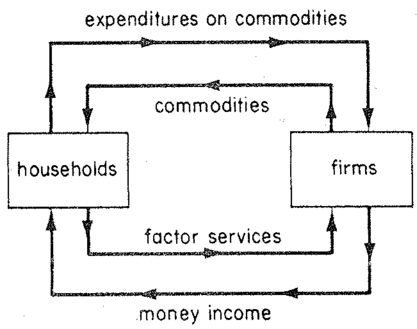

Figure 22.1 Circular flows in a two-sector economy

The *real flow* is the exchange of goods for the services of factors of production: firms produce and offer final goods to the household sector, and consumers offer to firms the services of factors which they own.

The *monetary flow* is the real flow expressed in monetary terms. The consumers receive income payments from the firms for offering their factor services. These incomes are spent by consumers for the acquisition of the finished goods produced by the business sector. The expenditures of firms become the money incomes of the households. Similarly, the expenditures of households become the receipts of firms, which they once again pay the households for the factor services which they supply.

The real flow and the monetary flow, which represent the transactions and the interdependence of the two sectors, move in opposite directions. They are linked by the prices of goods and factor services. The economic system is in equilibrium when a set of prices is attained at which the magnitude of the income flow from firms to households is equal to the magnitude of the money expenditure flow from households to firms.[3]

The interdependence of markets is concealed by the partial equilibrium approach.

[1] The government sector and the foreign sector are excluded from this simple model.

[2] This excludes the production of intermediate goods, i.e. goods produced by some firms and used by other firms as inputs.

[3] In fact, those two streams of payments represent the two traditional ways of measurement of an economy's total income. The payment of incomes by firms to households represents 'the income approach'. The payment of expenditures by households to firms represents the 'product approach'.

C. EXISTENCE, UNIQUENESS AND STABILITY OF AN EQUILIBRIUM

Three problems arise in connection with a general equilibrium:

1. Does a general equilibrium solution exist? (Existence problem.)
2. If an equilibrium solution exists, is it unique? (Uniqueness problem.)
3. If an equilibrium solution exists, is it stable? (Stability problem.)

These problems can best be illustrated with the partial-equilibrium example of a demand–supply model. Assume that a commodity is sold in a perfectly competitive market, so that from the utility-maximising behaviour of individual consumers there is a market demand function, and from the profit-maximising behaviour of firms there is a market supply function. An equilibrium exists when at a certain positive price the quantity demanded is equal to the quantity supplied. The price at which $Q_D = Q_S$ is the equilibrium price. At such a price there is neither excess demand nor excess supply. (The latter is often called *negative excess demand.*) Thus an equilibrium price can be defined as the price at which the excess demand is zero: the market is cleared and there is no excess demand.

The equilibrium is stable if the demand function cuts the supply function from above. In this case an excess demand drives price up, while an excess supply (excess negative demand) drives the price down (figure 22.2).

The equilibrium is unstable if the demand function cuts the supply function from below. In this case an excess demand drives the price down, and an excess supply drives the price up (figure 22.3).

In figure 22.4 we depict the case of multiple equilibria. It is obvious that at P_1^e there is a stable equilibrium, while at P_2^e the equilibrium is unstable. Finally in figure 22.5 an equilibrium (at a positive price) does not exist.

It should be clear from the above discussion that (*a*) the existence of equilibrium is related to the problem of whether the consumers' and producers' behaviour ensures

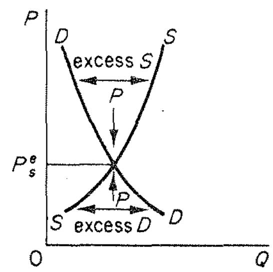

Figure 22.2 Unique, stable equilibrium

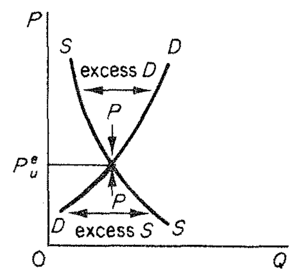

Figure 22.3 Unique, unstable equilibrium

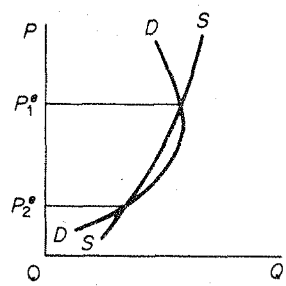

Figure 22.4 Multiple equilibria

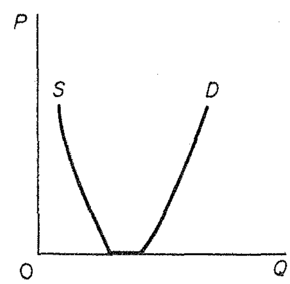

Figure 22.5 No equilibrium exists

that the demand and supply curves intersect (at a positive price); (*b*) the stability of equilibrium depends on the relationship between *the slopes* of the demand and supply curves; (*c*) the uniqueness of equilibrium is related to *the slope of the excess demand function*, that is, the curve which shows the difference between Q_D and Q_S at any one price.

In fact the three basic questions related to the existence, stability and uniqueness of an equilibrium can be expressed in terms of the excess demand function:

$$E_{(P_i)} = Q_{D(P_i)} - Q_{S(P_i)}$$

To see this we redraw below figures 22.2–22.5 in terms of the excess demand function. For each of these cases we have derived the relevant excess demand function by subtracting Q_S from Q_D at all prices.

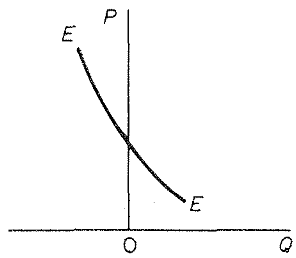

Figure 22.6 Stable equilibrium: slope of $E_{(P)} < 0$

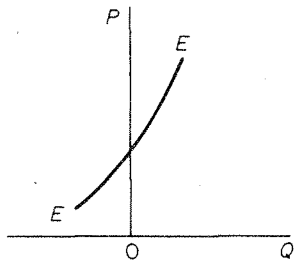

Figure 22.7 Unstable equilibrium: slope of $E_{(P)} > 0$

From the redrawn diagrams (in conjunction with the corresponding ones 22.2–22.5) we can draw the following conclusions.

1. The excess demand function, $E_{(P)}$, intersects the vertical (price)-axis when there is an equilibrium, that is, when the excess demand is zero. If $Q_D = Q_S$, then $E_{(P)} = 0$.

2. There are as many equilibria as the number of times that the excess demand curve $E_{(P)}$ intersects the vertical price-axis (figure 22.8).

3. The equilibrium is stable if the slope of the excess demand curve is negative at the point of its intersection with the price-axis (figure 22.6).

4. The equilibrium is unstable if the slope of the excess demand curve is positive at the point of its intersection with the price-axis (figure 22.7).

5. If the excess demand function does not intersect the vertical axis at any one price, an equilibrium does not exist (figure 22.9).

The above analysis of the existence, stability and uniqueness in terms of excess demand functions can be extended to general equilibrium analysis.[1]

[1] See E. Roy Weintraub, *General Equilibrium Theory*. Macmillan Studies in Economics (Macmillan, 1974).

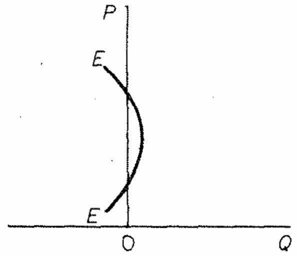

Figure 22.8 Multiple equilibria

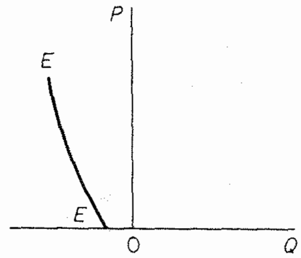

Figure 22.9 No equilibrium exists

D. A GRAPHICAL ILLUSTRATION OF THE PATH TO GENERAL EQUILIBRIUM

In this section we attempt to show how a simple economy with perfectly competitive production and factor markets will have an inescapable tendency toward a general equilibrium solution.

Assumptions

1. Two substitute commodities are produced, X and Y, by two perfectly competitive industries (markets).
2. There are two factors, capital K and labour L, whose markets are perfectly competitive. The quantities of these factors are given (fixed supply).
3. The production functions are continuous, with diminishing marginal rate of factor substitution and decreasing returns to scale.
4. The industry producing X is less capital intensive than the industry producing Y. The K/L ratio is smaller in industry X.
5. Consumers maximise utility and firms maximise profit.
6. The usual assumptions (of large numbers, homogeneous products and factors, and free entry and exit) of perfect competition hold.
7. The system is initially in equilibrium: in all markets demand is equal to supply at a positive equilibrium price.

Assume that an exogenous change in consumers' tastes shifts the demand for X outwards to the right, from D_0 to D_1, causing the price, in the short run, to rise from P_0 to P_1^x and the quantity sold to increase by X_0X_1 units (figure 22.10).

Since X and Y are substitutes (*ex hypothesi*)[1] one should expect the increase in the demand for X to be accompanied by a decrease in the demand for Y.[2] The demand for Y (figure 22.12) shifts to the left, its price drops and the quantity of Y sold decreases by Y_1Y_0 units. (Note that with the assumption of given K and L, the economy cannot produce simultaneously increased quantities of both X and Y. Hence we must assume that an increase in the demand for X is accompanied by a decrease in the demand for Y: there cannot be an increase in the demand for X without a corresponding concurrent decrease in the demand for Y, unless we allow for inflation.)

The increase in P_x creates excess profits for the producers of X (figure 22.11) and losses for the producers of Y (figure 22.13). Firms are thus induced to divert resources from the production of Y to the production of X. This is shown by the movement

[1] The two commodities cannot be complementary in the 2 × 2 × 2 model.

[2] Since the factors of production are given, a simultaneous increase of both X and Y cannot be dealt without complicating the analysis with inflationary phenomena.

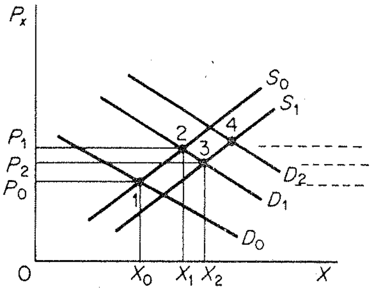

Figure 22.10 Industry X

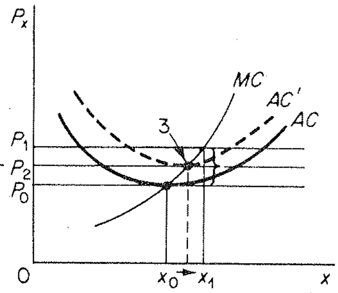

Figure 22.11 A firm in industry X

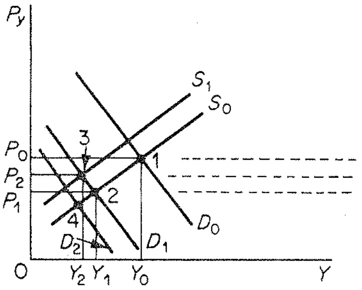

Figure 22.12 Industry Y

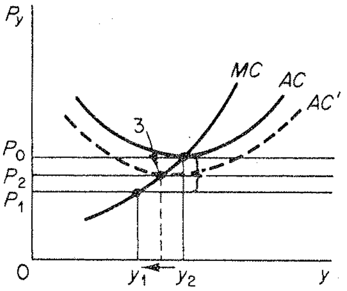

Figure 22.13 A firm in industry Y

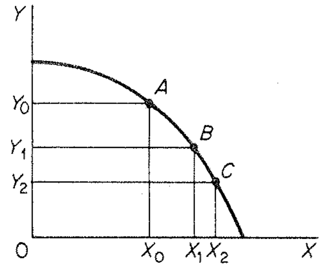

Figure 22.14 Production possibility curve

from point A to point B on the production possibility curve (figure 22.14). This shift reflects the effect of the change in consumers' tastes on the decisions of firms. The increase in P_x induces the producers of X to increase their quantity in order to maximise their profit, given the increase in marginal revenue. (The reaction of a typical firm in industry X is shown in figure 22.11.) Since every firm in the X industry faces the same price the output of each firm increases (each firm produces on the rising part of its AC). The sum of the increases in outputs of *existing* firms is equal to the increase $X_0 X_1$ in figure 22.10.

In industry Y the opposite occurs. The fall in P_y induces firms to decrease their quantity. The reaction of a typical firm in industry Y is shown in figure 22.13. The sum of the decreases in output of the individual firms is equal to the decline $Y_0 Y_1$ in figure 22.12.

In the long run excess profits attract entry in industry X and induce exit of firms from industry Y. Entry and exit affect the demand for factors of production. The markets for labour and capital used in the production of X are shown in figures 22.15 and 22.17.

The expansion of production by existing firms and the entry of new firms in industry X increases the demand for labour and capital. The D_{L_x} and D_{K_x} curves shift outwards and w_x and r_x rise. Employment of these factors rises in industry X (by $L_0 L_1$ and $K_0 K_1$ respectively in figures 22.15 and 22.17). The demand for L and K by a single firm in industry X is shown in figures 22.16 and 22.18. The individual firm can buy any quantity of L and K at the prevailing market price. At the increased price w_1^x the firm will demand l_1 of labour (figure 22.16), and at the increased price r_1^x the firm will demand k_1 of capital (figure 22.18).

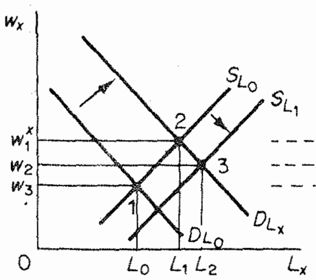

Figure 22.15
Labour market for industry X

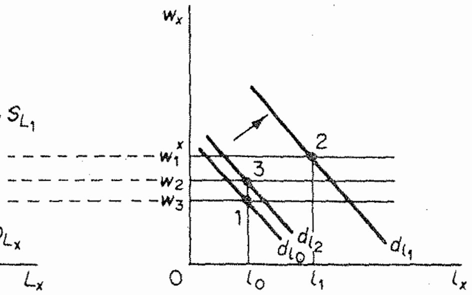

Figure 22.16 Demand for labour by a firm in industry X

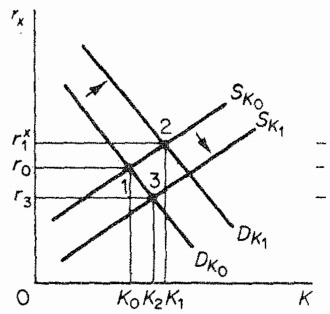

Figure 22.17 Market for capital in industry X

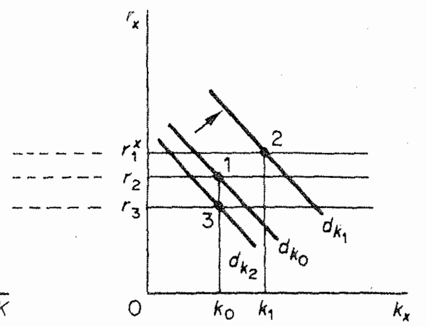

Figure 22.18 Demand for capital by a firm in industry X

The situation in the labour and capital markets for the industry Y will be the reverse. The surviving firms in this industry will reduce their demand for both factors, while the exit of firms in the long run will further reduce the demand for these inputs. The situation for the factor markets in industry Y is shown in figures 22.19 and 22.21. The case of a single firm in industry Y is depicted in figures 22.20 and 22.22.

The above change in the demands for factors in the two industries leads to disequilibrium, because the prices of L and K have risen in the factor markets for industry X, while w and r have fallen in the factor markets of industry Y. However, in perfect factor markets the disequilibrium will be self-correcting, since in the long run there is perfect mobility of factors between the different markets. Thus the owners of L and K will withdraw their services from the Y industry and will seek to have them employed by firms in the X industry where w and r are higher.

The above reactions (mobility) of factor suppliers will result in an upward shift of the supply curve of factors in industry Y and a downward shift of the supply curves of factors in industry X. The shifts are shown in figures 22.15, 22.17, 22.19 and 22.21.

With the assumption that X is more capital intensive than commodity Y the prices of factors will not return to their original levels. w and r will be equalised in the two industries, but the wage level will be higher in the new equilibrium while the price of capital r will be lower in the final equilibrium (this is shown by the point 3 in figures 22.15, 22.17, 22.19, 22.21). The assumption of different factor intensities $((K/L)_x < (K/L)_y)$ has the following repercussions. The demand of the X industry for labour is stronger than the demand for capital. The release of labour by industry Y is smaller than the rate required by X, while the release in capital is larger than the increased need for this factor by industry X. Thus overall the demand for labour will be higher than initially and w will rise. The opposite will occur in the market for capital, where the new equilibrium r will be lower than in the initial situation.

As a result of the new factor prices the individual firms in each industry will adjust their demands for labour and capital. In figures 22.16, 22.18, 22.20 and 22.22 the individual firms are in equilibrium at the points denoted by the digit (3).

The entry on firms in industry X shifts the supply curve downwards to S_1 (figure 22.10). The final equilibrium price of X is lower than in the short run, but higher than in the original equilibrium. The X industry is an increasing cost industry.

The exit of firms from industry Y shifts the supply of this industry upwards (figure 22.12). The final equilibrium price P_2 is higher than the short-run price P_1, but lower than the initial equilibrium level P_0. Industry Y is also an increasing cost industry.

Given the new product prices the individual firms will adjust (independently) their outputs. Firms in industry X will be producing an output lower than in the short run, but higher than in the initial equilibrium (figure 22.11). Firms in industry Y will be producing an output higher than in the short run, but lower than in the initial situation (figure 22.13). The changes in w and r cause the LAC of firms in X to shift

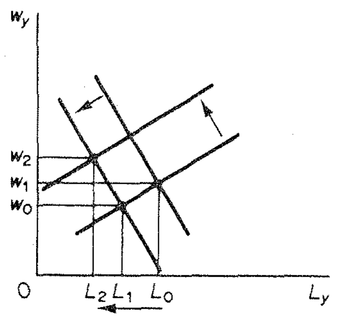

Figure 22.19
Labour market for industry Y

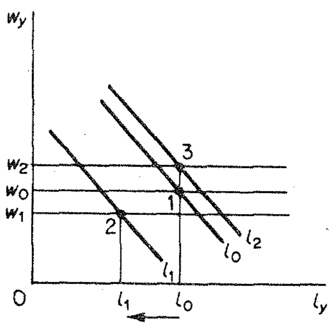

Figure 22.20 Demand for
labour by a firm in industry Y

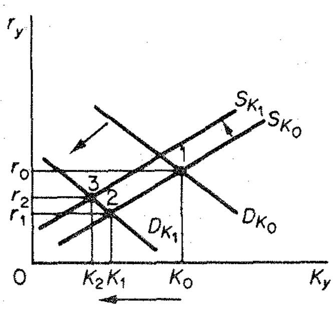

Figure 22.21 Market for
capital for industry Y

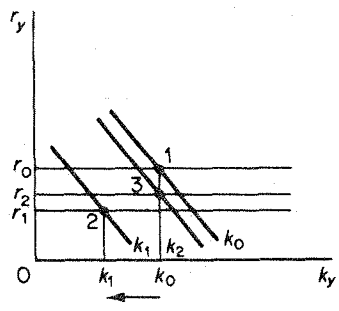

Figure 22.22 Demand for
capital by a firm in industry Y

2. Static properties of a general equilibrium state (configuration)

Three static properties are observed in a general equilibrium solution, reached with a free competitive market mechanism:

(*a*) Efficient allocation of resources among firms (equilibrium of production).

(*b*) Efficient distribution of the commodities produced between the two consumers (equilibrium of consumption).

(*c*) Efficient combination of products (simultaneous equilibrium of production and consumption).

These properties are called *marginal conditions of Pareto optimality* or *Pareto efficiency*. A situation is defined as Pareto optimal (or efficient) if it is impossible to make anyone better-off without making someone worse-off. The concept of Pareto optimality will be discussed in detail in Chapter 23. In the following paragraphs we discuss briefly the three optimality properties that are observed in a general equilibrium state.

(a) Equilibrium of production (efficiency in factor substitution)

Equilibrium of production requires the determination of the efficient distribution of the available productive factors *among the existing firms* (efficiency in factor substitution).

From Chapter 3 we know that the firm is in equilibrium if it chooses the factor combination (for producing the most lucrative level of output) which minimises its cost. Thus the equilibrium of the firm requires that

$$\begin{bmatrix}\text{slope of}\\ \text{isoquant}\end{bmatrix} = \begin{bmatrix}\text{slope of}\\ \text{isocost}\end{bmatrix}$$

or

$$MRTS_{L,K} = \frac{w}{r}$$

where w and r are the factor prices prevailing in the market and $MRTS$ is the marginal rate of technical substitution between the factors.

The joint equilibrium of production of the two firms in our simple model can be derived by the use of the Edgeworth box of production.[1] On the axes of this construct we measure the given quantities of the factors of production, $\bar{K}$ and $\bar{L}$ (figure 22.23). The isoquants of commodity X are plotted with origin the south-west corner and the isoquants of Y are plotted with origin the north-east corner. The locus of points of tangency of the X and Y isoquants is called the *Edgeworth contract curve of production.*[2] This curve is of particular importance because it includes the efficient allocations of K and L between the firms.

Each point of the Edgeworth box shows a specific allocation of K and L in the production of commodities X and Y. Such an allocation defines six variables: the

[1] A detailed description of the construction of the Edgeworth box of production is given in Chapter 3, p. 100.

[2] In constructing the Edgeworth box we have assumed that X is less capital intensive than Y. If the K/L ratio were the same for the two products the contract curve would be a straight line.

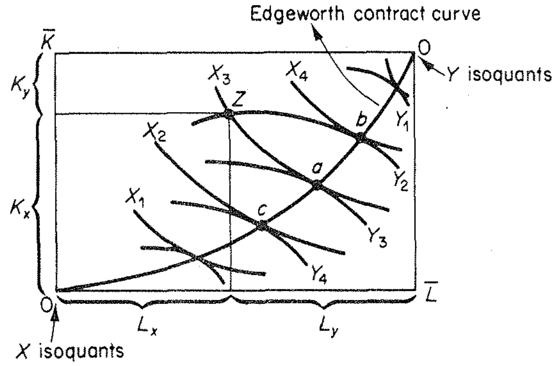

Figure 22.23 Edgeworth box of production

amounts of Y and X produced and the amounts of capital and labour allocated to the production of Y and X. For example point Z shows that:

X_3 is the quantity produced of commodity X
Y_2 is the quantity produced of commodity Y
K_x is the amount of capital allocated to the production of X_3
K_y is the amount of capital allocated to the production of Y_2
L_x is the amount of labour allocated to the production of X_3
L_y is the amount of labour allocated to the production of Y_2

However, not all points of the Edgeworth box represent efficient allocations of the available resources. Given that K and L are limited in supply, their use should produce the greatest possible output. An allocation of inputs is efficient if the produced combination of X and Y is such that it is impossible to increase the production of one commodity without decreasing the quantity of the other.[1] From figure 22.23 we see that efficient production takes place *on* the Edgeworth contract curve. It is impossible to move to a point off this curve without reducing the quantity of at least one commodity. Point Z is a point of inefficient production, since a reallocation of K and L between the two commodities (or firms) such as to reach any point from a to b leads to a greater production of one or both commodities.

Since the Edgeworth contract curve of production is the locus of tangencies of the X and Y isoquants, at each one of its points the slopes of the isoquants are equal:

$$\begin{bmatrix}\text{slope of} \\ X \text{ isoquant}\end{bmatrix} = \begin{bmatrix}\text{slope of} \\ Y \text{ isoquant}\end{bmatrix}$$

or

$$MRTS^x_{L,K} = MRTS^y_{L,K}$$

In our simple general equilibrium model the firms, being profit maximisers in competitive markets, will be in equilibrium only if they produce somewhere *on* the Edgeworth contract curve. This follows from the fact that the factor prices facing the

[1] The above definition of efficiency is also known as *Pareto efficiency* or *Pareto optimality* after the name of the Italian economist Vilfredo Pareto (*Cours d'Économie politique* (Lausanne, 1897)). This concept is further discussed in Chapter 23.

producers are the same, and their profit maximisation requires that each firm equates its $MRTS_{L,K}$ with the ratio of factor prices w/r:

$$MRTS^x_{L,K} = MRTS^y_{L,K} = \frac{w}{r} \quad (1)$$

In summary. The general equilibrium of production occurs at a point where the $MRTS_{L,K}$ is the same for all the firms, that is, at a point which satisfies the Pareto-optimality criterion of efficiency in factor substitution: the general equilibrium of production is a Pareto-efficient allocation of resources. The production equilibrium is not unique, since it may occur at any point along the Edgeworth contract curve: there is an infinite number of possible Pareto-optimal production equilibria. However, with perfect competition, one of these equilibria will be realised, the one at which the ('equalised' between the firms) $MRTS_{L,K}$ is equal to the ratio of the market factor prices w/r. That is, with perfect competition general equilibrium of production occurs where condition (1) is satisfied.

If the factor prices are given, from the Edgeworth box of production we can determine the amounts of X and Y which maximise the profits of firms. However, in a general equilibrium, these quantities must be equal to those which consumers want to buy in order to maximise their utility. Consumers decide their purchases on the basis of the prices of commodities, P_x and P_y. Thus, in order to bring together the production side of the system with the demand side, we must define the equilibrium of the firms in the product space, using as a tool the *production possibility curve* of the economy.[1] This is derived from the Edgeworth contract curve of production, by mapping its points on a graph on whose axes we measure the quantities of the final commodities X and Y. From each point of the Edgeworth contract curve of production we can read off the maximum obtainable quantity of one commodity, given the quantity of the other. For example, point a in figure 22.23 shows that, given the quantity of X is X_3, the maximum quantity of Y that can be produced (with the given factors $\bar{K}$ and $\bar{L}$) is Y_3. The X_3, Y_3 combination is presented by point a' in figure 22.24. Similarly, point b of the Edgeworth contract curve of production shows that, given X_4, the maximum amount of Y that the economy can produce is Y_2. Point b' in figure 22.24 is the mapping of b from the factor space to the production space.

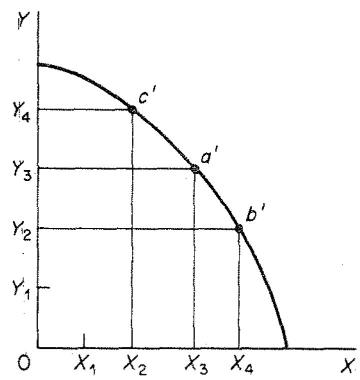

Figure 22.24 Production possibility curve

[1] The production possibility curve has been introduced in Chapter 3 in relation to the equilibrium of a multi-product firm.

and

$$dK_x = -dK_y \tag{4a}$$

Substituting (4) in (3) we find

$$\frac{d(TC_x)}{d(TC_y)} = \frac{w\cdot(-dL_y) + r\cdot(-dK_y)}{w\cdot(dL_y) + r\cdot(dK_y)} = -1 \tag{5a}$$

(5) Finally, substituting (5) in (2), we obtain

$$\frac{MC_x}{MC_y} = (-1)\frac{dY}{dX} = -\frac{dY}{dX} = (\text{slope of } PPC) = MRPT_{x,y} \qquad \text{Q.E.D.}$$

In perfect competition the profit-maximising producer equates the price of the commodity produced to the long-run marginal cost of production:

$$MC_x = P_x \quad \text{and} \quad MC_y = P_y$$

Therefore the slope of the production possibility curve is also equal to the ratio of the prices at which X and Y will be supplied by perfectly competitive industries:

$$MRPT_{x,y} = \frac{MC_x}{MC_y} = \frac{P_x}{P_y} \tag{2}$$

Given the commodity prices, general equilibrium of production is reached at the point on the production transformation curve that has a slope equal to the ratio of these prices. Such a general equilibrium of production is shown in figure 22.25. Assume that the market prices of commodities define the slope of the line AB. The ratio $0A/0B$ measures the ratio of the marginal cost of and hence the supply price of X to that of Y.

The general equilibrium product-mix from the point of view of firms is given by point T. The two firms are in equilibrium producing the levels of output Y_e and X_e.

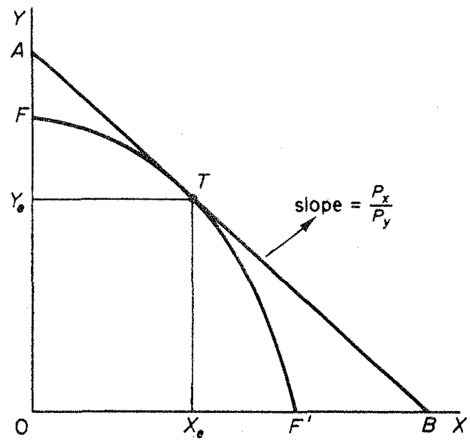

Figure 22.25 General equilibrium of production with perfect competition

(b) Equilibrium of consumption (efficiency in distribution of commodities)

We must now show how each consumer, faced with the market prices P_x and P_y, reaches equilibrium, that is, maximises his satisfaction. From the theory of consumer behaviour (Chapter 2) we know that the consumer maximises his utility by equating the marginal rate of substitution of the two commodities (slope of his indifference curves) to the price ratio of the commodities. Thus the condition for consumer equilibrium is

$$MRS_{x,y} = \frac{P_x}{P_y}$$

Since both consumers in perfectly competitive markets are faced with the same prices the condition for joint or general equilibrium of both consumers is

$$MRS^A_{x,y} = MRS^B_{x,y} = \frac{P_x}{P_y} \quad (3)$$

This general equilibrium of consumption for the product mix Y^e, X^e is shown in figure 22.26. We construct an Edgeworth box for consumption with the precise dimensions Y^e and X^e by dropping from point T (on the product transformation curve) lines parallel to the commodity axes. We next plot the indifference curves of consumer A with origin the south-west corner, and the indifference curves of B with origin the north-east corner.

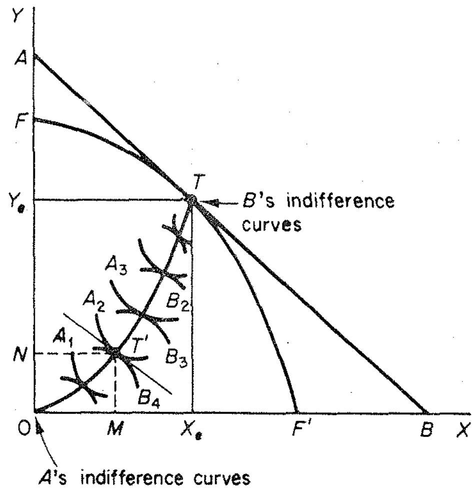

Figure 22.26

Any point in the Edgeworth consumption box shows six variables: the total quantities Y^e and X^e, and a particular distribution of these quantities between the two consumers. However, not all distributions are efficient in the Pareto sense. A Pareto-efficient distribution of commodities is one such that it is impossible to increase the utility of one consumer without reducing the utility of the other.[1] From figure 22.26 it is seen that only points of tangency of the indifference curves of the two consumers represent Pareto-efficient distributions. The locus of these points is called the *Edgeworth contract curve of consumption*. It should be clear that at each point of this curve the following equilibrium condition is satisfied

$$MRS^A_{x,y} = MRS^B_{x,y}$$

[1] See also Chapter 23.

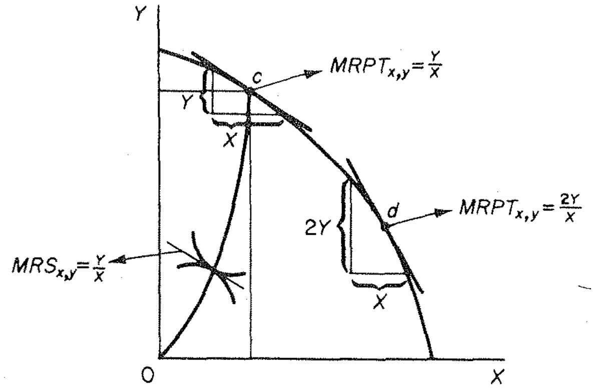

Figure 22.27 $MRPT_{x,y} = 2Y/X$, $MRS_{x,y} = Y/X$. Disequilibrium of production and consumption

In summary, with perfect competition (and no discontinuities and with constant returns to scale) the simple two-factor, two-commodity, two-consumer system has a *general equilibrium solution*, in which three Pareto-efficiency conditions are satisfied:

1. The *MRS* between the two goods is equal for both consumers. This *efficiency in distribution* implies optimal allocation of the goods among consumers.
2. The *MRTS* between the two factors is equal for all firms. This *efficiency in factor substitution* implies optimal allocation of the factors among the two firms.
3. The *MRS* and the *MRPT* are equal for the two goods. This *efficiency in product-mix* implies optimal composition of output in the economy and thus optimal allocation of resources.[1]

Whether such a general equilibrium solution (on the *PPC*) is desirable for the society as a whole is another question, which is the core of Welfare Economics. In the next chapter we will be concerned with the significance for the economic welfare (of the society as a whole) of the existence of a general equilibrium solution reached with perfect competition.

3. General equilibrium and the allocation of resources

In figure 22.26 the general equilibrium solution is shown by points T (on the production possibility curve) and T' (on the Edgeworth contract curve). These points define six of the 'unknowns' of the system, namely the quantities to be produced of the two commodities (X_e and Y_e), and their distribution among the two consumers (X_e^A, X_e^B, Y_e^A, Y_e^B). In this section we examine the determination of the allocation of resources between X and Y. The determination of the remaining unknowns (prices of factors and commodities, and the distribution of income between the two consumers) is examined in two separate sections below.

Point T on the production transformation curve (figure 22.26) defines the equilibrium product mix Y_e and X_e. Recalling that the *PPC* is the locus of points of the Edgeworth contract curve of production mapped on the product space, point T corresponds to a given point on this contract curve, say T'' in figure 22.28. Thus T'' defines the allocation of the given resource endowments in the production of the

[1] Because in perfect competition all three marginal conditions for Pareto-optimal resource allocation are satisfied, perfect competition is considered as an 'ideal' market structure, in the sense that the scarce resources are used in the most efficient way.

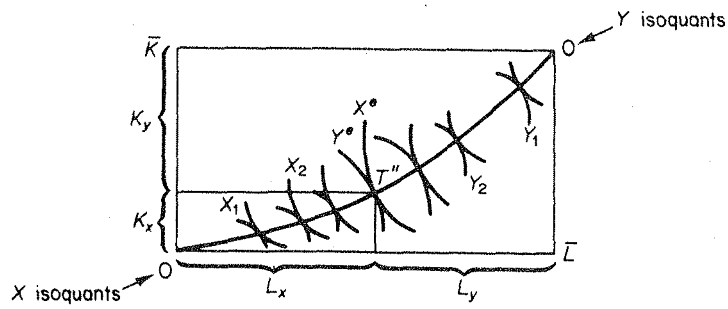

Figure 22.28 Allocation of resources to the production of X_e and Y_e

general equilibrium commodity mix. The production of X_e absorbs L_x of labour and K_x of capital, while Y_e employs the remaining quantities of factors of production, L_y and K_y. Thus four more 'unknowns' have been defined from the general equilibrium solution.

4. Prices of commodities and factors

The next step in our analysis is to show the determination of prices in the general equilibrium model, under perfect competition.

In the simple 2 × 2 × 2 model there are four prices to be determined, two commodity prices, P_x and P_y, and two factor prices, the wage rate w, and the rental of capital r. We thus need four independent equations.[1] However, given the assumptions of the simple model, we can derive only three independent relations.

1. Profit maximisation by the individual firm implies least-cost production of the profit-maximising output. This requires that the producer adjusts his factor mix until the *MRTS* of labour for capital equals the w/r ratio:

$$MRTS^x_{L,K} = \frac{w}{r} = MRTS^y_{L,K} \tag{5}$$

In other words the individual producer maximises his profit at points of tangency between the isoquants and isocost lines whose slope equals the factor price ratio.

2. In perfect factor and output markets the individual profit-maximising producer will employ each factor up to the point where its marginal physical product times the price of the output it produces just equals the price of the factor

$$w = (MPP_{L,x})\cdot(P_x) = (MPP_{L,y})\cdot(P_y) \tag{6}$$

$$r = (MPP_{K,x})\cdot(P_x) = (MPP_{K,y})\cdot(P_y) \tag{7}$$

3. The individual consumer maximises his utility by purchasing the output mix which puts him on the highest indifference curve, given his income constraint. In other words maximisation of utility if attained when the budget line, whose slope is equal to the ratio of commodity prices P_x/P_y, is tangent to the highest utility curve, whose slope is the marginal rate of substitution of the two commodities

$$MRS^A_{y,x} = \frac{P_y}{P_x} = MRS^B_{y,x} \tag{8}$$

[1] Note that the graphical analysis of the general equilibrium solution required the *ratios* of the input prices. The *absolute values* of prices were of no importance in our simple model, where the equilibrium of firms and consumers is attained by equating ratios of prices to various transformation (of commodities) and substitution (of factors) ratios.

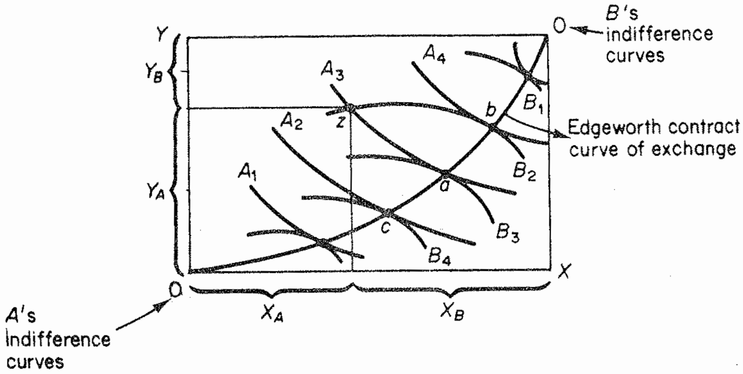

Figure 23.1 Edgeworth box of exchange

distribution of goods between consumers. This curve is formed from the points of tangency of the two consumers' indifference curves, that is, points where the slopes of the indifference curves are equal. In other words, at each point of the contract curve the following condition is satisfied

$$MRS^A_{x,y} = MRS^B_{x,y}$$

Therefore we may state the marginal condition for a Pareto-efficient distribution of given commodities as follows:

The marginal condition for a Pareto-optimal or -efficient distribution of commodities among consumers requires that the MRS *between two goods be equal for all consumers.*

(*b*) Efficiency of allocation of factors among firm-producers

To derive the marginal condition for a Pareto-optimal allocation of factors among producers we use an argument closely analogous to the one used for the derivation of the marginal condition for optimal distribution of commodities among consumers. In the case of allocation of given resources K and L we use the Edgeworth box of production which we explained in detail in Chapter 22. Such a construct is shown in figure 23.2.

Only points on the contract curve of production are Pareto-efficient. Point H is inefficient, since a reallocation of the given K and L between the producers of X and Y such as to reach any point from c to d inclusive results in the increase of at least one commodity without a reduction in the other.

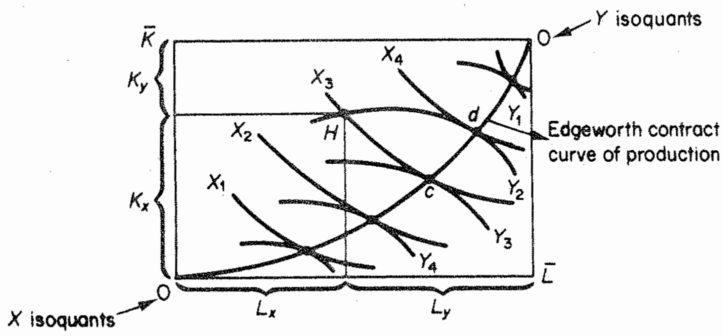

Figure 23.2 Edgeworth box of production

(situations, configurations) in which different individuals enjoy different utility levels. If the economy consists of two individuals the social welfare function could be presented by a set of social indifference contours (in utility space) like the ones shown in figure 23.3. Each curve is the locus of combinations of utilities of *A* and *B* which yield the same level of social welfare. The further to the right a social indifference contour is, the higher the level of social welfare will be. With such a set of social indifference contours alternative states in the economy can be unambiguously evaluated. For example a change which would move the society from point *b* to point *c* (or *d*) increases the social welfare. A change moving the society from *a* to *b* leaves the level of social welfare unaltered.

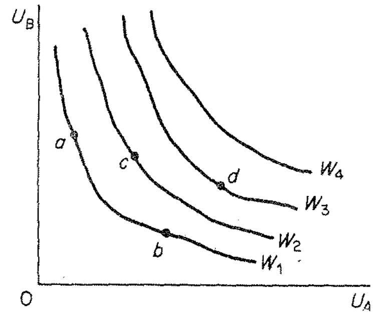

Figure 23.3 Bergson's welfare contours

The problem with the social welfare function is that there is no easy method of constructing it. Its existence is axiomatically assumed in welfare economics (see below). Somebody in the economy must undertake the task of comparing the various individuals or groups and rank them according to what he thinks their worthiness is. A democratically elected government could be assumed to make such value judgements which would be acceptable by the society as a whole. This is implicitly or explicitly assumed when use is made of the apparatus of the social welfare function.

It should be noted that the social welfare function cannot be used to derive social (or community) indifference curves in output space (analogous to the indifference curves of a single individual) without taking into account the distribution of income among the various individuals in the economy. In a subsequent section we will examine the conditions under which community indifference curves in output space can be derived from the social welfare function.[1]

B. MAXIMISATION OF SOCIAL WELFARE

In this section we will examine the conditions of social welfare maximisation in the simple two-factor, two-commodity, two-consumer model. The assumptions of our analysis are listed below.

1. There are two factors, labour *L*, and capital *K*, whose quantities are given (in perfectly inelastic supply). These factors are homogeneous and perfectly divisible.

[1] P. A. Samuelson, 'Social Indifference Curves', *Quarterly Journal of Economics* (1956), pp. 1–22.

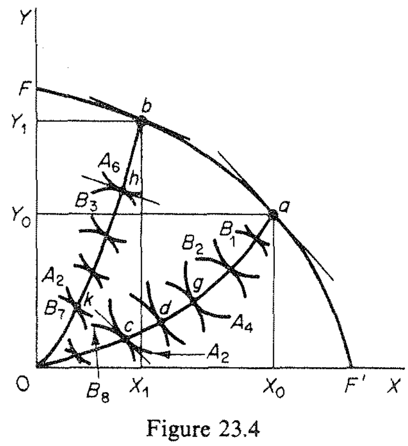

Figure 23.4

We may repeat this process for any other distribution of the given $Y_0 X_0$ product-mix. For example, at point g in figure 23.4 the two consumers enjoy the utilities A_4 and B_3. This utility combination is shown by point g' in the utility space of figure 23.5. Point g in output space maps into g' in utility space. Point g' shows the maximum utility attainable to B (B_3) given the utility of A (A_4) from the distribution of $Y_0 X_0$ denoted by point g in figure 23.4. By mapping all the points of the contract curve $0a$ into corresponding points in the utility space we obtain the utility possibility frontier for the particular commodity-mix $Y_0 X_0$ (curve SS' in figure 23.5). The utility possibility curve SS' is drawn for the specific product-mix $Y_0 X_0$, and it shows maximum utility possibilities when the economy produces this specific combination of commodities. Since there is an infinite number of points on the *PPC* curve, there must be an infinite number of utility possibility curves, each such curve for each product-mix on the production possibility curve. For example, assume that the economy produces the output-mix $Y_1 X_1$ denoted by point b on the *PPC* curve of figure 23.4. The points on the Edgeworth contract curve $0b$ show Pareto-optimal distributions of the product-mix $Y_1 X_1$. Point h shows the utility combination $A_6 B_3$, which is depicted by point h' in the utility space of figure 23.5. Similarly, point k in output space is mapped into point k' in utility space. The remaining points of the $0b$ contract curve are mapped into the points of the utility possibility frontier RR' in figure 23.5.

In summary, each point on the *PPC* gives rise to a utility possibility frontier. The *envelope* of these utility possibility frontiers is *the grand utility possibility frontier* of the economy.

There is an alternative way of deriving the grand utility possibility frontier, which is much simpler. It makes use of the third marginal condition of Pareto optimality, that the slope of the *PPC* be the same as the 'equalised' *MRS* of the two commodities for the two consumers ($MRPT_{x,y} = MRS^A_{x,y} = MRS^B_{x,y}$). For any commodity combination produced in the economy, such as a on the *PPC* in figure 23.4, we pick the point on the corresponding contract curve ($0a$) which has the same slope as the *PPC* at a. At this point of the contract curve (c in figure 23.4) $MRPT = MRS$. From the utilities of consumers A and B associated with c we can obtain c' in the utility space of figure 23.5; c' shows the utilities of A and B when the product-mix $Y_0 X_0$ (denoted by a) is distributed between them in a way satisfying the Pareto-optimality condition $MRPT = MRS$. Point c' shows the maximum or *grand utility* attainable to the society from the output combination $Y_0 X_0$. Thus only point c' of the SS' utility frontier belongs to the 'envelope' (or 'grand') utility possibility frontier.

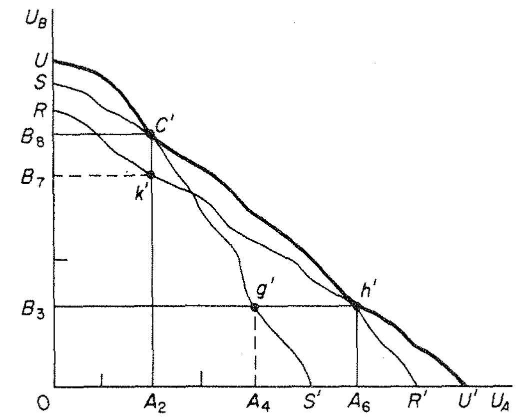

Figure 23.5 The grand utility possibility frontier

For each point on the *PPC* we can repeat the above procedure and obtain a point in utility space. For example, if the product-mix is *b*, point *h* on the corresponding 0*b* contract curve indicates the optimal distribution of this product-mix between the two consumers. The 'grand' utility associated with this distribution is represented by point *h'* in the utility space of figure 23.5. Repetition of this procedure for each point on the *PPC* yields the 'envelope' utility possibility frontier. This is shown by the curve *UU'* in figure 23.5. Thus, the grand utility possibility frontier is the locus of utility combinations (of the two consumers) which satisfy the marginal condition $MRPT = MRS$ for each commodity-mix. Each point of the 'envelope' shows the maximum of *B*'s utility for any given feasible level of *A*'s utility, and vice versa.

It should be clear from the above discussion that all points on the grand utility possibility frontier satisfy all the Pareto marginal conditions of efficiency: efficiency in production, efficiency in distribution, efficiency in product composition.

2. DETERMINATION OF THE WELFARE-MAXIMISING STATE: THE 'POINT OF BLISS'

In figure 23.6 the grand utility possibility frontier is combined with the social welfare function shown by the set of social indifference contours.

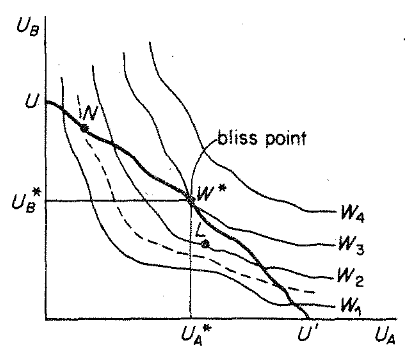

Figure 23.6 Maximisation of social welfare

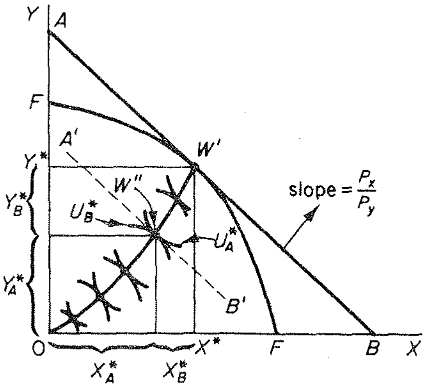

Figure 23.7

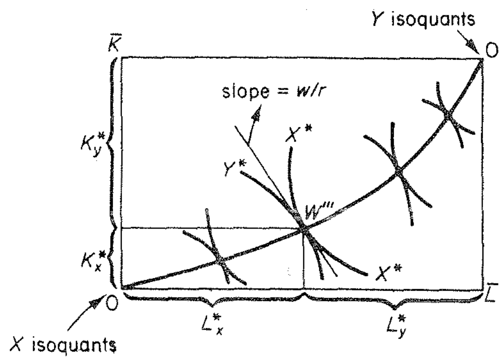

Figure 23.8 Optimal allocation of resources

which will be used in the production of Y^* and X^*. The welfare-maximising resource allocation is shown by the length of the segments marked by the brackets and denoted by the symbols L_x^*, L_y^*, K_x^* and K_y^*.

In summary, the maximum-welfare configuration is determinate. We have solved (found unique values) for the total outputs, the distribution of these outputs between the two consumers, and for the labour and capital to be used in the production of the welfare-maximising outputs.[1]

[1] An interesting consequence of the concave shape of the *PPC* is that the *value* of the product-mix that maximises social welfare, estimated at the 'shadow-prices' embeded in the system is at a maximum. For the definition of the 'prices' implied by the solution of the welfare-maximisation problem, see below, p. 537). An examination of figure 23.9 makes this clear. The line *AB*, whose slope is the ratio of the 'prices' of the commodities, P_x/P_y, can be thought of as an 'isovalue' line. The equation of the line *AB* is derived from the relation

$$V = P_x \cdot X + P_y \cdot Y$$

where V = total value of the output.

Solving for Y we obtain the *AB* line

$$Y = (1/P_y) \cdot V - (P_x/P_y) \cdot X$$

Given the prices, we can form a family of isovalue curves, by assigning different values to V. The higher the line, the greater the value of the total output will be.

D. WELFARE MAXIMISATION AND PERFECT COMPETITION

We have demonstrated that under certain assumptions an economy can reach the point of maximum social welfare. It should be stressed that the bliss point (and the solution of the system for the values of the ten variables that are the unknowns in the welfare-maximisation problem of the $2 \times 2 \times 2$ model) depends only on technological relations: the problem of welfare maximisation is purely 'technocratic'. Recall that the bliss point is attained by equalising the slopes of isoquants, the slopes of the indifference curves, and the slope of the production possibility curve to the (equalised) slope of the indifference curves. Thus, the welfare-maximising solution does not depend on prices. However, in Chapter 22 we have shown that perfect competition can lead to a general equilibrium situation where the three marginal conditions of Pareto optimality are satisfied. The analysis of Chapter 22 can now be extended to show that the general equilibrium solution reached with perfect competition is the same as the situation implied by the bliss point of the welfare maximisation problem.

(*a*) Profit maximisation by the individual firm implies that whatever output the firm may choose as the most profitable must be produced at a minimum cost. Cost minimisation is attained if the firm chooses the input combination at which the marginal rate of technical substitution of the two factors is equal to the input price ratio

$$MRTS_{L,K} = \frac{w}{r}$$

Since in perfect competition all firms are faced by the same set of factor prices, it follows that

$$MRTS^x_{L,K} = MRTS^y_{L,K} = \frac{w}{r}$$

Thus in figure 23.8 the slope of the contract curve at W''' must be equal to w/r in perfectly competitive input markets, since W''' is the point of the (equalised) slope of the isoquants Y^* and X^*.

(*b*) Utility maximisation by each individual requires the choice of the product-mix where the marginal rate of substitution of the two commodities is equal to the ratio

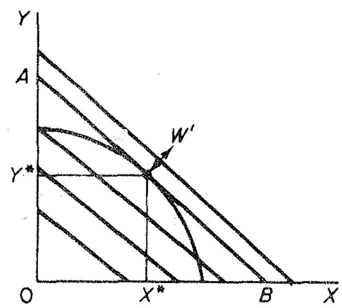

Figure 23.9

Now, the welfare-maximising commodity-mix is defined by the point of tangency of the production possibility curve with the line *AB*, which is the highest possible isovalue line in our example. Hence, at the price ratio implied by the line *AB*, point *W'* defines the welfare-maximising quantities of the two commodities, and at the same time the highest output-value.

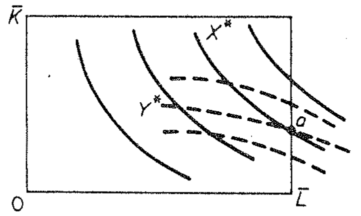

Figure 23.10

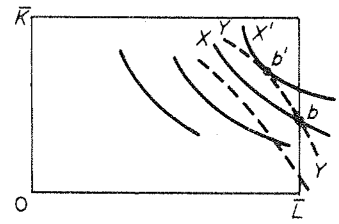

Figure 23.11

slope of the Y isoquant, so that the 'inequality' equilibrium condition is not satisfied.

In summary, for a corner solution to occur the Y isoquant must cut (on the axis of the Edgeworth box) the X isoquant from below.

3. EXISTENCE OF COMMUNITY INDIFFERENCE CURVES IN OUTPUT SPACE

The bliss point, where the social welfare function was maximised, was defined in *utility space*, that is, on the graph on whose axes we measure the utilities of the two consumers (in ordinal indexes), while the utility-maximising position of an individual consumer is determined in *output space* (i.e. on a graph on whose axes we measure the quantities of the two commodities). The social welfare contours are *not* social or community indifference curves (equivalent to a single individual's indifference curves). For a *single individual*, a given XY combination (in output space) belongs to a unique indifference curve and has a unique slope ($MRS^A_{x,y}$). However, if the particular XY combination is distributed between *two consumers*, a community indifference curve passing through XY (in output space) would not have a unique slope, because this slope would be sensitive to the way that the product mix XY is distributed among the two individuals. Recall that a particular product-mix can be distributed optimally among A and B in an infinite number of ways, along the contract curve of the Edgeworth box of exchange corresponding to this product mix. Each distribution (of the given product-mix) has a different (equalised) MRS for the two individuals, because it corresponds to a different utility combination. These utility combinations, corresponding to the different distributions, form the utility possibility frontier for the particular commodity-mix. Accordingly, the community MRS at a given point in commodity space (i.e. the slope of a community indifference curve) will vary with movements along the corresponding utility possibility frontier (i.e. with the distribution of the XY combination between A and B).

In summary, we can say that a point in output space maps to a *curve* in utility space; and a point in utility space maps into a *curve* in output space. Not just one, but many possible XY combinations can yield a specified $U_A U_B$ mix. It is this reciprocal point-line phenomenon that lies at the heart of Samuelson's proof of the non-existence of community indifference curves. The community MRS for a given fixed Y and X combination depends on how X and Y are distributed among A and B, i.e. on which $U_A U_B$ point on the Edgeworth contract curve of exchange is chosen. Hence the slope of a community indifference curve for a given XY mix is not uniquely determined.

However, if one can decide which is the most desirable $U_A U_B$ combination for a given 'basket' of X and Y, then the equalised MRS of the two individuals at this utility combination can be considered as the unique MRS of the community as a whole, so that the community indifference curve at the XY point will have a unique slope. Based on this, Samuelson[1] proved that one can derive community indifference curves by continuous redistribution of 'incomes' until the welfare function (axiomatically assumed to exist) is maximised in utility space. To illustrate this assume an initial distribution (point *a* in figure 23.12) which yields a total welfare of W_0 (point *a'* in figure 23.13). With continuous redistribution the community can reach point *e* on W_2 where the social welfare is maximised for the particular $Y_0 X_0$ output mix. (We saw that *e* is mapped into a *single* point in output space.)

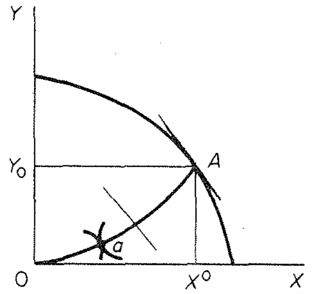

Figure 23.12

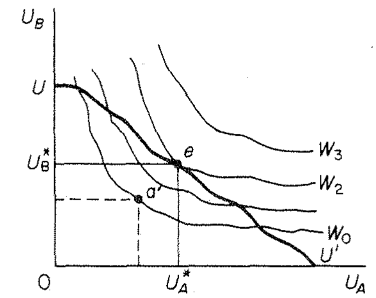

Figure 23.13

If we repeat this process for all output combinations, we can derive a set of *social indifference curves in the output space.* Note the two basic assumptions of Samuelson's proof of 'existence' of social indifference curves: (*a*) A social welfare function exists; (*b*) continuous redistribution of 'incomes' is possible.

In summary, the maximisation of social welfare procedure can be used to 'establish' the existence of community indifference curves in output space. These curves are called 'community indifference curves *corrected for income distribution*'. They are widely used in international trade theory and in other fields of economics. Bator (op. cit.) argues that these community indifference curves reveal the society's ranking of preferences, as reflected in a social welfare function which is defined by a government elected by political consensus. Bator believes that the rankings of the society, based on these community indifference curves is more objective than that of any 'arbitrary ethic standard', adopted subjectively by policy-makers. We think that this argument is 'circular', since the community indifference curves (in output space) are derived from a social welfare function which is assumed to exist axiomatically, and which incorporates the subjective value judgements of those who supposedly have defined it. The point is that, given the subjective nature of the welfare function, the community indifference curves will also reflect subjective valuations.

4. ELASTIC SUPPLY OF FACTORS

The 2 × 2 × 2 model assumed given quantities of factors. This assumption can be relaxed. The supply of factors can be expressed as a function of all the prices in the system. This allows for some elasticity in the supply of factors. The analytical effect is to make the *PPC* a function of the factors of production. This approach has been adopted in the generalised H × M × N model, presented in the Appendix to Chapter 22.

[1] See P. A. Samuelson, 'Social Indifference Curves', *Quarterly Journal of Economics* (1956) pp. 1–22.

of an action to the society as a whole. In other words, externalities create a divergence between private and social costs and benefits. Because externalities are not reflected in market prices, these prices provide misleading information (signals) for an optimal allocation of resources.

We will examine separately the externalities in production and the externalities in consumption.

A. Externalities in production

(a) Divergence between private and social costs

We will illustrate this case with an example. Assume that commodity X is alcohol, which for simplicity we assume is manufactured in a perfectly competitive market. Each firm is in equilibrium when

$$MC_x = P_x$$

where MC_x is the cost of the individual firm, or the private cost. This does not include the cost of pollution of the environment that the firm creates, nor the costs of accidents and deaths caused by drunken consumers. These are external costs to the firm. Suppose that the Health Department obtains an estimate of these costs. The marginal social cost (MSC) is the sum of the private cost (MC_x) and the marginal external cost (MEC), that is

$$MSC_x = MC_x + MEC$$

Obviously we have a divergence of private and social cost of alcohol. Since $MC_x = P_x$, it follows that $P_x < MSC_x$, which implies that the allocation of resources to the production of alcohol is not socially optimal; since the firm does not pay the full cost the production of alcohol in the economy is excessive. If the firm were made to pay the full social costs it would produce a smaller amount of alcohol, defined by the point where

$$P_x = MSC_x$$

This is shown in figure 23.14. The marginal social cost curve lies above the private marginal cost curve, given $MSC_x > MC_x$. The vertical difference between these two curves is the marginal external costs incurred in the production and consumption of alcohol. If the firm does not pay the external costs, its profit-maximising output is X_0. However, if the government required the firm to pay the external costs the firm would reduce its output to X_1.

In summary, when the private cost is smaller than the social cost, adherence to the rule $MC_x = P_x$ leads to overproduction of X. By an analogous argument it can be

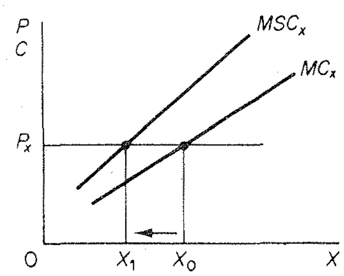

Figure 23.14

shown that if $MC_x > MSC_x$, the level of production of X will be less than the socially optimum. Divergence between private and social costs (and benefits) results to misallocation of resources in a perfectly competitive system.

(b) Divergence between price and social benefit

Even if the price is equal to the MSC there is no guarantee that social welfare is maximised, because price may be different than the social benefit. For example assume that an environmentalist uses unleaded petrol for running his car, paying $P_g = MC_g$. By using unleaded gasoline the environmentalist keeps the air cleaner, thus creating a benefit to others who breathe in a less polluted atmosphere. Since they do not pay for this benefit we have an externality for the society as a whole. If we add the value of this benefit to the price we obtain the marginal social benefit (MSB). Apparently $P_g < MSB_g$, and since the firm produces where $P_g = MC_g$, it follows that the production of gasoline is less than the socially optimal quantity. If the government added the external benefit on the price of petrol, the consumers would pay the full MSB_g, and each firm would increase its production. This is shown in figure 23.15.

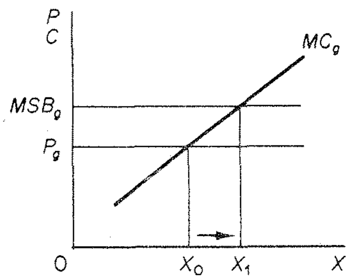

Figure 23.15

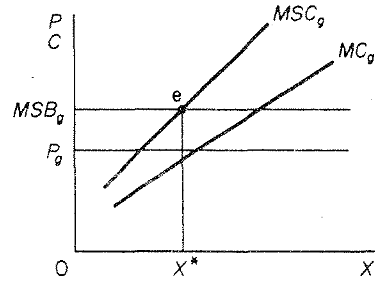

Figure 23.16

The MSB curve is above the P_g curve at all levels of output. If consumers pay the full MSB_g, the firm would increase its output by the amount X_0X_1. If we take into account any *external costs* of lead-free petrol the marginal cost curve would shift to the left (figure 23.16) and equilibrium would be reached at point *e*, where

$$MSC_g = MSB_g$$

From the above discussion we may conclude that when externalities exist, the condition for a socially optimal production is the equality of the MSC and the MSB. In a multi-product economy the condition for optimal resource allocation is

$$\frac{MSB_x}{MSC_x} = \frac{MSB_y}{MSC_y} = \cdots = \frac{MSB_M}{MSC_M} = 1$$

(c) External economies in production

We said that the presence of externalities in production invalidates the conditions required for social welfare maximisation. The question is how important are externalities in the real world. Some examples may illustrate the extent of the problem.

(i) A new highway reduces the transport cost of individual firms. Since they do not pay for the construction of the highway the MSC is higher than the private marginal cost.

(ii) The expansion of an industry (for example the motorcar industry) creates additional demand for the industries that supply it with raw materials, intermediate products and machinery. This increase in demand in the other industries may allow

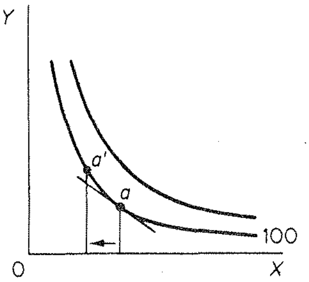

Figure 23.17
A's indifference map

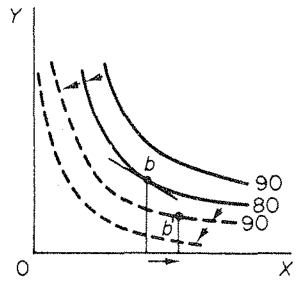

Figure 23.18
B's indifference map

consumption of commodity X (for example tobacco), but not of Y (for example alcohol). Under these conditions a redistribution of the same total output of X and Y between the two consumers can increase social welfare. Assume that redistribution is such as to decrease the consumption of X by consumer A. This *shifts* the utility map of B downwards, showing the increase in the utility of B arising from the reduction in the consumption of X by consumer A. Redistribution is such that consumer A moves to point a' and consumer B moves to b'. The total welfare has increased, despite the fact that at the new equilibrium the $MRS_{x,y}$ is *not* the same for the two consumers. In the new situation consumer A is on the same indifference curve, while B has moved to a higher indifference curve (point b' lies on the shifted indifference curve with utility 90). We conclude that, when externalities in consumption exist, adherence to the equalisation of the MRS of the two consumers does not ensure Pareto optimality.

8. KINKED ISOQUANTS

We have assumed that the isoquants are continuous and smooth, without kinks. Such smooth curvatures are mathematically convenient, because calculus can be applied in the solution of the problem of welfare maximisation. Kinked isoquants cause indeterminacy in marginal rates of substitution, that is, lead to a breakdown of calculus techniques. However, kinked isoquants can be handled with the linear programming techniques. The mathematics become more complicated, but the model retains its essential properties. All the efficiency conditions can be restated so as to take into account the kinked isoquants. Furthermore the existence of implicit 'prices' embedded in the maximum-welfare problem is, if anything, even more striking in linear programming.[1]

9. CONVEX ISOQUANTS

We have assumed that the isoquants are convex to the origin and there are constant returns to scale. These assumptions ensure the concavity of the production possibility curve, which is essential for the solution of the welfare-maximisation problem. In this paragraph we examine the effects of the relaxation of the assumption of convexity of the production isoquants. In the next section we will examine the effects of increasing returns to scale.

[1] See R. Dorfman, P. A. Samuelson, and R. Solow, *Linear Programming and Economic Analysis* (McGraw-Hill, 1958).

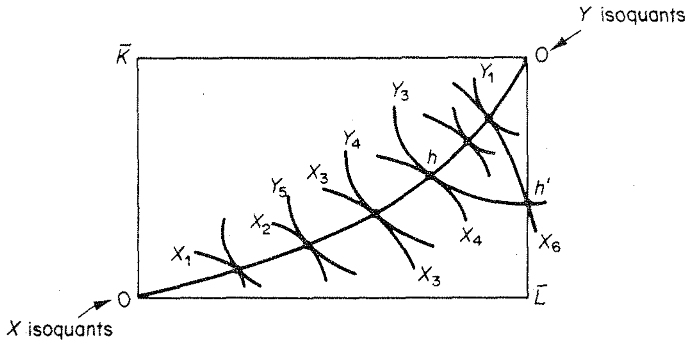

Figure 23.19

If the production isoquants are concave to the origin the Edgeworth contract curve of production becomes the locus of tangencies (of the isoquants of X and Y) where the output of X is *minimum* for a given level of Y, and vice versa. Consider figure 23.19, where both sets of isoquants are concave. Any point on the contract curve FF' shows the minimum quantity of X for a given quantity of Y. For example, point h shows the combination $Y_3 X_4$. However, given Y_3 the maximum amount of X is X_6 (at a 'corner solution'), while X_4 shows the minimum amount of X given Y_3.

We conclude that with concave isoquants the welfare-maximisation condition that the marginal rate of technical substitution of X and Y be equalised ($MRTS^x_{L,K} = MRTS^y_{L,K}$) will result in input combinations that give a minimum of one commodity for specified amount of the other, that is, a configuration which does not maximise social welfare.

10. INCREASING RETURNS TO SCALE

Returns to scale are related to the *position* of the isoquants, *not* to their shape. Assume that the isoquants are convex. Increasing returns to scale are shown by isoquants that are closer and closer (for output levels which are multiples of the original level) along any ray from the origin: to double output the firm needs less than double inputs. (See Chapter 3.)

In our simple model we assumed constant returns to scale. We also saw that decreasing returns to scale do not create serious difficulties except, perhaps, with the treatment of the imbalance between total value of the product and total payments to the factors (the product is not 'exhausted' by factor payments) within a general equilibrium approach. However, increasing returns to scale lead to serious difficulties. In this section we will examine the effects of increasing returns to scale (*a*) on the average cost curves of the firm, and (*b*) on the curvature of the production possibility curve.

(a) Increasing returns to scale and the AC curves of the firm

The consequence of increasing returns to scale is that the LAC falls as output increases. From Chapter 4 we know that when the AC curve falls, the MC curve lies below it. This situation is shown in figure 23.20.

The condition for profit maximisation in a perfectly competitive market is that the firm sets its MC equal to the market price

$$MC_x = P_x$$

However, when there are increasing returns to scale, adherence to this rule would

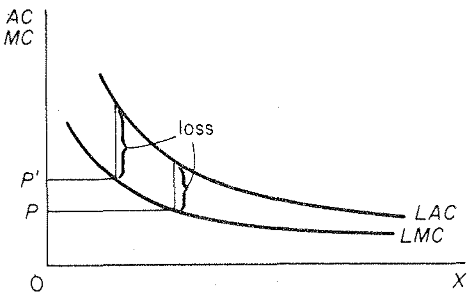

Figure 23.20

lead to losses, since $MC < AC$. In other words the maximisation of social welfare requires $P_x < AC$, that is, perpetual losses. But this situation is incompatible with perfect competition: firms which have losses close down in the long run, and the market system collapses.

Increasing returns to scale create another problem, namely the total value of the product is less than total factor payments

$$(w \cdot L + r \cdot K) > (P_x \cdot X + P_y \cdot Y)$$

(b) Increasing returns to scale and the shape of the production possibility curve

We will continue to assume that the factor intensity (K/L ratio) is different in the two commodities. In particular, the way in which we have been drawing the contract curve of production implies that the $(K/L)_x$ is less than the $(K/L)_y$.[1] This can be seen from figure 23.21. At point a we have $(K/L)_x < (K/L)_y$. Similarly at point b we observe that $(K/L)_x$ is less than $(K/L)_y$. The shape of our contract curve implies that X is less capital intensive than Y.

With different factor intensities, if the increasing returns to scale are not important, the PPC can still be concave to the origin, so that the model is valid. While doubling the inputs in the production of X would more than double the output of X, an increase in X at the expense of Y will, in general, not take place by means of such proportional expansion of factors, because efficient production takes place along the contract curve, and at each point of this curve the K/L ratio of X and Y change; as

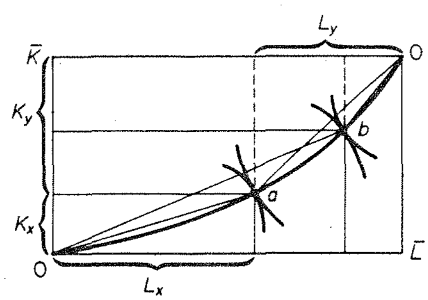

Figure 23.21

[1] If the K/L ratio were the same for both commodities the contract curve of production would coincide with the diagonal. In this case, with the assumption of constant returns to scale, the PPC would be a straight line with negative slope.

we have drawn the contract curve of production, labour is more important relative to capital in producing X, and vice versa for Y. Thus as the production of X expands, the producers of X will be coerced to use capital in greater proportion to labour. In other words, the K/L ratio in X becomes less 'favourable' as the production of X expands. The opposite is true of the K/L ratio in Y as the production of Y declines.

The above argument remains valid if we have unimportant increasing returns to scale in both functions. The *PPC* will become less curved, but so long as it stays below the diagonal of the Edgeworth box the production possibility curve will be concave, as required for the unique solution of the welfare-maximisation problem. In this 'mild' case of increasing returns to scale, with a still concave *PPC*, the previous maximising rules give the correct result for a maximum social welfare. Furthermore, the constants embedded in the system retain their meaning as 'prices', because they still reflect marginal rates of substitution and transformation. In this case also, the total value of maximum-welfare 'national' output ($V^* = P_x \cdot X + P_y \cdot Y$), valued at these 'shadow prices' (constants of the system), is still at a maximum.

If, however, the increasing returns to scale are strong the production possibility curve will be convex to the origin (figure 23.22). In this case two results are possible, depending on the curvature of the community indifference curves.

(i) If the curvature of the community indifference curves is greater than the curvature of the convex production possibility curve, social welfare is maximised at e, where the isovalue–product line AB is tangent to the production possibility curve. The constants implied by the maximum-of-welfare problem retain their meaning as 'shadow prices': they still reflect marginal rates of substitution and transformation. However, maximum social welfare is no longer associated with maximum value-of-output. In fact at point e the value-of-output is at a a minimum. Given P_x/P_y the value of output would be maximised at either F or at F' (the 'corners' of the convex production possibility curve).[1] Thus, if the curvature of the *PPC* is smaller than that of the community indifference curves, social welfare is maximised, but the value of output is at a minimum.

(ii) If, however, the curvature of the *PPC* is greater than the curvature of the community indifference curves, both social welfare and value of output are minimised if we apply the rules of welfare maximisation. In figure 23.23 the production possibility curve FF' is more convex than the indifference curves (U), and the point of tangency z is a point of both minimum welfare and minimum value of output. Welfare maximisation is attained at point F, a 'corner tangency' of the *PPC* and the highest possible community indifference curve (U_2).

In summary: When the *PPC* is convex to the origin, the *relative* curvatures (of the

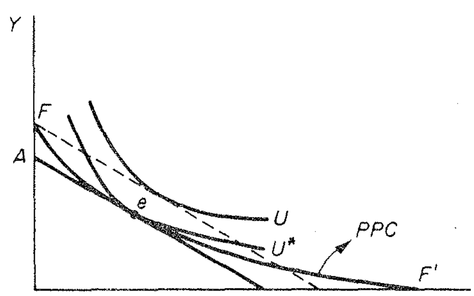

Figure 23.22

[1] In figure 23.22 the slope of AB is such that the value of output would be maximised at F.

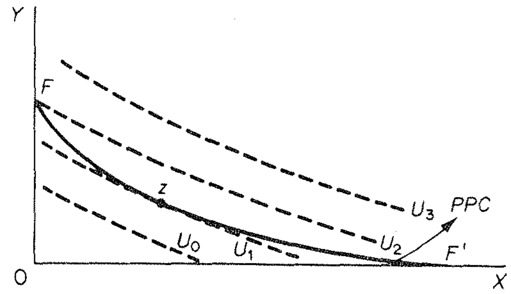

Figure 23.23

PPC and the community indifference curves) are crucial: tangency points may be either maxima or minima of welfare.

11. INDIVISIBILITIES IN THE PRODUCTION FUNCTION

If the technology consists of small-scale and large-scale production processes, with large-scale methods having a lower average cost than the small-scale methods, but large-scale methods are indivisible, then perfect competition does not result in a Pareto-optimal allocation of resources, nor to a maximisation of social welfare.

Assume a situation in which perfect competition prevails with a large number of small firms being in equilibrium. Furthermore, assume that the economy has attained the equality of the *MRPT* and the (equalised) *MRS* of the two commodities among the consumers. Although in this situation (configuration) the three marginal conditions of Pareto optimality are fulfilled, the use of resources is inefficient and social welfare is not maximised if the production methods are indivisible and the small firms cannot take advantage of the lower cost of the large-scale production techniques. Under these conditions it is obvious that a few large firms, using the more efficient large-scale methods, can produce a greater amount of output with the same total quantities of inputs available in the economy. This means that the small firms produce at a point below the *PPC*, because, due to the indivisibilities, they cannot make full use of the available technical knowledge. Although the small firms satisfied the marginal condition for efficient production the use of resources was really inefficient in the initial situation: the small firms produced less output with the same amounts of resources.

In conclusion, the existence of indivisibilities is incompatible with the assumptions of perfect competition and the 2 × 2 × 2 welfare model.

From the above examination of the assumptions of the 2 × 2 × 2 model, we may conclude that the model collapses when:

(i) a welfare function does not exist;
(ii) there are externalities in production;
(iii) there are interdependencies in the utility functions;
(iv) there are strong economies of scale, which render the *PPC* convex to the origin *and* its curvature is greater than the curvature of the community indifference curves;
(v) there are indivisibilities in the production function.